Journal of Engineering and Applied Sciences

Volume: 1
Edition: 1
Year: 2014
Pages: 1–12

Reliability of Using Curvature Techniques in Structural Damage Detection

Mohamed Abdel-Basset Abdo

Department of Civil Engineering, Faculty of Engineering, Al-Jouf University,
P.O. Box 2014 Al-Jouf-Skaka, Saudi Arabia, mohd.abdo2002@yahoo.com

Abstract

A damage in a structure alters its dynamic characteristics; namely, natural frequencies, modal damping values, and mode shapes. Changes also occur in some of the structural parameters such as: the mass, damping and stiffness matrices of the structure. Among different algorithms developed for structural damage identification, changes in either mode shape curvature or uniform load surface (ULS) curvature, derived from measured modal properties, have shown promise for locating structural damage. However, to date there is not a study reported in the technical literature that directly compares these two promising methods. The numerical results in this paper attempt to fill this void in the study of damage detection methods. In this paper, a numerical study is investigated to compare the robustness of these two methods in damage detection using the frequencies and mode shapes of the first few modes. Also, the application of a damage localization algorithm to these two methods in detecting and locating damage is demonstrated. The numerical results show that both of the two methods can accurately locate single damage with different damage characteristics (location and severity). However, the two methods have shown less sensitivity to specific types of damage when applied to multiple damage locations. Also, the results of the ULS curvature method contain less noise than that of the results of the mode shape curvature method. Finally, the mode shape curvature can pinpoint damage locations even with one of the lower mode shapes of the structure and it does not require the mode frequency.

Keywords: Structural Damage Detection; Mode shape Curvature; Uniform Load Surface Curvature; Dynamic Characteristics.
Article history: Received 31 Dec 2013, Accepted 4 Feb 2014

1. Introduction

Interest in the ability to monitor a structure and detect damage at the earliest possible stage is pervasive throughout the civil, mechanical, and aerospace engineering communities. Current damage detection methods in civil structures are either visual or localized experimental methods such as; acoustic emission or ultrasonic methods, electro-magnetic field methods, X-ray, radiographs, eddy current methods, and thermal field methods, Doebling *et al*., 1996. All of these experimental techniques have some limitations such as: 1) the quality of the process is often dependent on the inspection personnel experience and knowledge and 2) results from one (local) area of a structure does not necessarily represent conditions at another area. Furthermore, there is a possibility that damage could be undetected at inspection or that growth of cracks in load-carrying members to critical levels, for instance, could occur between inspection intervals and lead to a structure collapse. So, the need for more robust global damage monitoring systems have motivated investigators to study the application of global, vibration-based damage detection methods that can be applied to complex structures using changes in the global dynamic characteristics of the structure.

Over the past four decades detecting damage in a structure from changes in global dynamic parameters has received considerable attention from civil, aerospace and mechanical engineering communities. The basis for this approach is that changes in the structure physical properties such as: boundary conditions, stiffness, mass and/or damping will alter the structure dynamic characteristics; namely, natural frequencies, modal damping values, and mode shapes. Doebling, *et al.*, 1996 provided an excellent review on research advances in this area, and summarized this kind of technology as vibration-based damage identification methods.

Significant work has been done in the formulation of vibration-based damage detection algorithms. Early works make use of the natural frequency and mode shape information. Salawu, 1997 gave a literature review of the state of the art of damage detection using changes in natural frequency. Salawu and Williams, 1995 formulated changes in modal assurance criteria (MAC) and changes in co-ordinate modal assurance criterion (COMAC) between the intact and damaged structure as indicators to localize damage. Abdo and Hori, 2001 illustrated with theoretical and numerical analysis that the derivatives of mode shapes are good indicators of damage. Pandey *et al*. 1991 demonstrated that changes in mode shape curvature could be a good indicator of damage. Furthermore, Quan and Weiguo, 1998 showed that for a steel deck of a bridge, the curvature of mode shape is the best among three damage recognition indices based on mode shapes, (the COMAC, the flexibility, and the curvature of mode shape). In addition, they found that some first vibration mode shapes, whether vertical or horizontal modes, could be used equally to detect damage in the steel deck. Ratcliffe, 1997 successfully used a finite difference Laplacian function, which represents the curvature of the mode shape, to identify the location of damage as little as about 10 percent in a uniform beam. Unfortunately, this method had a problem for less severe damage because it needed further processing and gave smearing location of damage. Applications of modal curvature to concrete structures were found to be promising, Maeck and De Roeck, 1999, and recently by Pirner and Urushadze 2001. Abdo and Abdou, 2005 have investigated the robustness of using curvatures of high-order mode shapes (which can be measured by advanced sensors) in damage identification. They found that both low- and high-order mode shapes gave successful results even though the low-order mode shapes gave smoother localization than that of the high-order mode shapes. Abdo, 2012 investigated the application and reliability of using high-order mode shape derivatives especially, the fourth derivative in damage detection of plate-like structures. The results showed that the fourth derivative of mode shape is promising in detecting and locating structural damage in plate-like structures. Unfortunately, damage detection using changes in fourth derivative of mode shapes is sensitive to measurement noise.

During the last two decades some researchers found that the modal flexibility can be a more sensitive parameter than natural frequencies or mode shapes alone for structural damage detection. Raghavendrachar and Aktan, 1992 examined the application of modal flexibility for a three span concrete bridge. In their comparison with natural frequency and mode shapes, the modal flexibility is reported to be more sensitive and reliable for local damages. Zhao and Dewolf, 1999 presented a theoretical sensitivity study comparing the use of natural frequencies, mode shapes, and modal flexibility for structural damage detection. The results demonstrate that modal flexibility is more likely to indicate damage than either the other two. Pandey and Biswas, 1994 proposed a damage localization method based on directly examining the changes in the measured modal flexibility of a beam structure. Also, Pandey and Biswas, 1995 presented an experimental verification of locating damage in structures using flexibility difference method. Lu *et al., 2002* pointed out that Pandey's method is difficult to locate multiple damages, and they recommended the modal flexibility curvature for multiple damage localization due to its high sensitivity to closely distributed structural damages. Zhang and Aktan, 1995 studied the modal flexibility and its derivative, called uniform load surface (ULS) and stated that the change in curvature of the uniform load surface can be used to determine the location of damage. Wu and Law, 2004 extended the application of uniform load surface curvature to plate structures and showed that the ULS has much less truncation effect and is less sensitive to experimental errors. These features make the ULS curvature a potentially useful index for experimental nondestructive evaluation.

So far among different algorithms developed for damage identification, changes in mode shape curvature and uniform load surface curvature, derived from measured modal properties, have shown promise for locating structural damage. However, to date there has not been a study reported in the technical literature that directly compares these two successful methods. The numerical results reported in this paper attempt to fill this void in the study of damage detection methods. The objective of this paper is to clarify and understand the relative merits as well as the shortcomings of these two methods. A careful numerical study is carried out by using the finite element method to analyze dynamic behaviour of damaged structural members of a two-span continuous steel beam.

2. Theoretical background

2.1. Mode shape curvature

Let us consider a simply supported beam element which has the stiffness $K=EI$ with E and I being the modulus of elasticity and the second moment of the beam cross section, respectively. A narrow zone of damage with width h is assumed at $x_d - h/2 < x < x_d + h/2$ with the stiffness $K - \Delta K$.

As h goes to zero, the natural frequency as well as the associated displacement, slope, bending moment, and shear force approach those of the intact beam. However, the curvature at the intersection suffers a jump; for instance, at $x_d - h/2$, the curvature satisfies $M + h\Delta M = K\kappa^l = (K - \Delta K)\kappa^r$, where κ and M stand for the curvature and bending moment. By approximating M as $K\kappa^0(x_d)$, then the jump of the curvature is evaluated as:

$$[\kappa] \approx \left(\frac{K}{K \ - \Delta K} - 1 \right) \ \left| \kappa^0 (x_d) \right|. \tag{1}$$

Since h is small, this jump leads to a spike in the curvature of mode shape, its height is related to ΔK and κ^0 . The width of the spike is h and the height of the spike becomes sharper as h decreases and the height increases as ΔK increases. Thus, a reduction of stiffness associated with damage will, in turn, lead to an increase in curvature. Differences in the pre- and post-damage mode shape curvature will be largest in the damaged region. For multiple modes, the absolute values of change in mode shape curvature associated with each mode can be assumed to yield a damage parameter for a particular location.

2.2. Uniform load surface curvature

For a structural system with n degrees-of-freedom, the flexibility matrix can be expressed by superposition of the mass normalized modes ϕ_r as follows:

$$F = \sum_{\mathrm{r=1}}^{\mathrm{n}} \frac{\phi_r \phi_r^T}{\omega_r^2}, \tag{2}$$

where ω_r is the r th natural frequency. It can be seen from Eq. (2) that the modal contribution to the flexibility matrix decreases rapidly as the frequency ω_r increases, so the flexibility matrix converges rapidly as the number of contributing lower modes increases. This observation provides a great possibility to approximate closely the flexibility matrix with several lower modes.

In practice, there are only several, in most cases two to three lower vibration modes of a structure which can be obtained with confidence from modal testing. When m lower modes are available, the modal flexibility matrix can be approximated as follows:

$$F \approx \sum_{\mathrm{r=1}}^{\mathrm{m}} \frac{\phi_r \phi_r^T}{\omega_r^2} = \left[f_{k,l} \right], \tag{3}$$

in which the modal flexibility, $f_{k,l}$, at the k th point under the unit load at point l is the summation of the products of two related modal coefficients for each available mode,

$$f_{k,l} = \sum_{\mathrm{r=1}}^{\mathrm{m}} \frac{\phi_r(k)\phi_r(l)}{\omega_r^2}. \tag{4}$$

For a linear system, the modal deflection at point k under uniform unit load all over the structure can be approximated as

$$u(x) \approx \sum_{\mathrm{r=1}}^{\mathrm{m}} \frac{\phi_r(k) \sum_{l=1}^{n} \phi_r(l)}{\omega_r^2} = \sum_{l=1}^{n} f_{k,l}. \tag{5}$$

The uniform load surface (ULS) is defined as the deflection vector of the structure under uniform load.

$$U(t) = \{u(k)\} = F.V, \tag{6}$$

where $V = \{1,1,\ldots 1\}_{1\times n}^T$ is the unit vector representing the uniform load acting on the structure. From Eqs. (4) and (5), it is observed that the ULS converges more rapidly with the lower modes than the modal flexibility. This is because of the summation of all the modal coefficients of each mode to the ULS in Eq. (5). Since the modal coefficients of higher modes tend to cancel each other more than those of the lower modes, the lower modes tend to contribute more than the higher modes to the ULS coefficients. This canceling effect does not exist with the modal flexibility formulation in Eq. (4). These significant properties make the ULS a potentially stable and sensitive damage indicator for structural health monitoring.

2.3. Curvature based on central difference method

It is assumed that the dynamic response of the structure is acquired by placing sensors in a rectangular grid, so that the mode shapes, and then the ULS can be estimated. In the absence of damage, the mode shapes and ULS of the structure is a smooth surface over the loading plane. When there is a fault, sharp changes in the ULS, like a peak or abrupt slope, will appear at the fault location. Based on the study of damage detection with mode shapes and flexibility for beamlike structures (Pandey *et al.*, 1991; Lu *et al.*, 2002), the curvature technique is proven to be most efficient to locate these changes in the smooth curves.

So far all the reported studies on curvature-based damage detection computed the curvatures using a finite central differentiation procedure. Using this technique, the curvatures of the mode shapes or ULS are calculated by a Laplacian operator along the sensor grid as

$$u_i'' = \frac{u_{i+1} - 2u_i + u_{i-1}}{L^2}, \tag{7}$$

where L is the length of the element and the grid is assumed to be equally spaced.

If two sets of measurements, one from the intact structure and another from the damaged structure, are taken and the modal parameters are estimated from the measurements, the curvatures of mode shapes and ULS at any point for the two states can be obtained using Eq. (7). The presence of an irregularity in the damaged curvature can be detected by subtracting the curvature of the intact state from the curvature of the damaged state. If the structure is undamaged when the second set of measurement is carried out, the difference between the two sets will be the noise only. Therefore, values of the absolute differences of curvatures slightly oscillate around zero without any distinct peak. In contrast, spikes will clearly show up at the damaged zone of the beam, as will be shown hereafter.

2.4. Damage localization algorithm

We have known that the derivatives of mode shapes or ULS are sensitive and localized at the damaged region. Now, we need to make a decision whether a structure is undamaged or damaged at a given location. In this study, the algorithm which is developed by Stubbs *et al*., 1995, is used to calculate the damage indices that are function of measurable pre- and post-damage modal characteristics. The damage localization is accomplished in three steps as follows: 1) compute the absolute differences of the curvature at the *i* th node of the *j*th mode, β_{ij} , 2) normalize the values of the indicators according to the rule

$$I_{ij} = \left(\beta_{ij} - \mu_{\beta j}\right)/\sigma_{\beta j} \tag{8}$$

where, $\mu_{\beta j}$ and $\sigma_{\beta j}$ are respectively, the mean and standard deviation of the damage indices for the *j* th mode, 3) classify the damage location at any point, *i*. The damaged location is that at which the damage indices have values $I_{ij} \geq 2$. It should be mentioned that the points that have damage indices, $I_{ij} \geq 2$ indicate that the probability of false alarm (*Pfa*) is just 0.0228.

3. Numerical example

To accomplish the comparative study between the curvatures of mode shapes and ULS in detecting damage in structural systems, a two-span continuous steel beam is studied. The beam is assumed to have uniform cross sectional area and supported on a hinge support in the middle and roller supports at both ends. The pre- and post-damage modal parameters are calculated numerically using the software package MARC/Mentat, 2010 (a, b). Two-node beam element (element 98) with six degrees of freedom per node is used. The finite element model of the beam consists of 60 equal-length 2-D beam elements and 61 nodes as shown in Fig. 1. The cross sectional area

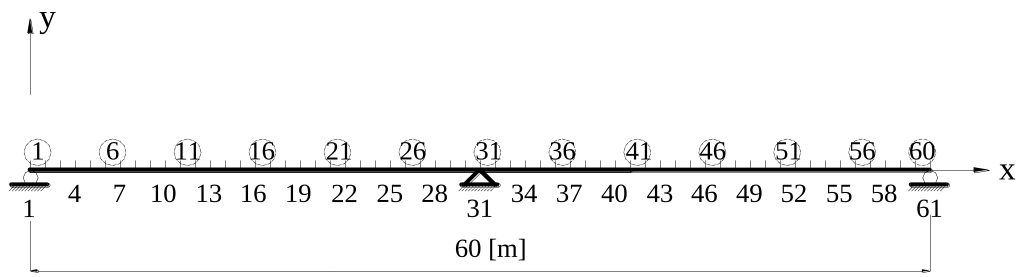


Fig. 1. Geometry of the two-span continuous beam

of the beam and the moments of inertia are, A=0.07 [m^2], and I_z= 0.040 [m^4], I_y= 0.001 [m^4], respectively, and the mechanical properties are, modulus of elasticity, E=210 GN/m^2, Poisson's ratio, ν=0.3, and the density, ρ=7,850 [kg/m^3].

The eigenmodes calculated are orthogonal with respect to the inertia matrix M, i.e., $x_i^T M x_i = 1$. Only the translational displacement mode in the y direction (Uy) is considered in the analysis. This was done because, in any experimental work, in general, rotations are not measured because of the difficulty in their measurements. Moreover, since we are interested only in the flexural modes of vibration, translation along the X axis can also be neglected. For the sake of comparison and to be applicable to experimental data, the displacement modes are normalized with respect to the square root of sum of squares, (SRSS).

The damage of the model in this study is assumed to affect only the stiffness matrix but not the inertia matrix in the eigenproblem formulation. This assumption is consistent with those used by Pandey *et al.*, 1991, Lu *et al*., 2002 and Wu and Law, 2004. The change in the stiffness due to damage is modeled by a reduction in the modulus of elasticity of the element. The degree of damage is then related to the extent of reduction of the modulus of elasticity, E, of some elements. A linear modal analysis is performed to examine the robustness of curvature techniques in damage detection.

Six cases of damage are studied. These cases of damage are investigated to represent not only different locations but also different severities of damage. For each case of damage, the first five natural frequencies and the corresponding mode shapes are calculated. Table 1 shows the damage characteristics of the six cases of damage. The first three cases of damage represent single damage with 30% reduction in modulus of elasticity, *E,* with different locations in elements No. 1, 8 and 15, respectively. The fourth and fifth cases of damage represent single damage with different severities of damage, 10% and 30% reduction in E of element 30. The sixth case of damage represents multiple damage locations with 30% reduction in E in elements No. 1, 8, 15 and 30, simultaneously. The four elements No. 1,8, 15, and 30 are chosen to simulate different types of damage scenarios. Damage in element No. 1 represents a failure due to shear at the end support where the displacement and curvature have minimum values. Damage in element No. 8 represents a failure due to combined shear and bending moment where the displacement and curvature have small values. Damage in element No. 15 represents a failure due to bending moment where the displacement and curvature have maximum values. Damage in element No. 30 represents a failure due to combined shear and bending moment where the displacement has minimum value and the curvature has maximum value. The first five cases of damage represent single damage location with different degrees of severities of damage, whereas *Case-6* is studied to show the capability of the damage detection methods to detect and pinpoint multiple damage locations.

4. Analysis of results

The first five frequencies and mode shapes for the intact and damaged beam models are investigated for the pre-mentioned six cases of damage. The displacement mode shapes are normalized by the square root of sum of squares (SRSS). As mentioned before, these six cases of damage represent different damage characteristics, (location and severity).

Table 1. Damage characteristics of the beam model

Case of damage	Damage location (Element No.)	Percentage reduction in E	Number of damage locations
Case-1	1	30	1
Case-2	8	30	1
Case-3	15	30	1
Case-4	30	10	1
Case-5	30	30	1
Case-6	1,8,15,30	30	4

In case of the uniform load surface curvature, first we calculate the participation of each of the first five modes to the uniform load surface (ULS) from the normalized displacement modes in the Y direction. Next, we calculate the ULS using Eq. (6) using the available natural frequencies and mode shapes. Then, we calculate the curvature of the ULS using the central difference approximation using Eq. (7). Then, the absolute differences of the curvatures of ULS are calculated. On the other hand, for damage detection using mode shape curvature, only the normalized mode shapes are needed. The curvatures of mode shapes are calculated directly using Eq. (7). The absolute differences of the curvatures between damaged and undamaged beam models are estimated for each of the first five modes. As expected, the absolute differences of the mode shape curvatures or ULS curvatures have clear spike(s) at the damaged element(s) for all scenarios of damage.

Now, the next problem is to classify the curvatures into damaged and undamaged locations. This can be accomplished by applying the damage localization algorithm using Eq. (8) to the absolute differences of curvatures. On calculating the damage indices of the curvatures, one can distinguish the damaged and undamaged element(s) of the beam model. The damaged nodes are those which have damage indices, $I \geq 2$.

5. Numerical results and discussions

5.1. Natural frequencies

The percentage changes in the first five natural frequencies of the two-span beam for different cases of damage are listed in Table 2. It is clear from this table that there is a discernible change in the natural frequencies between the intact and damaged beam, but this does not give an indication of the location of damage. This is expected because natural frequencies represent the dynamic characteristics of the whole structure. However, it is important to mention that the decrements of natural frequencies are very small for all modes of damage C*ase 1*. This is attributed to the fact that the node lines of these modes pass through the region of damage (element No. 1). The same thing can be said to the third and fourth modes in damage *Case 3* and to the first, third and fifth modes of damage *Case 4* and *damage Case 5*. It can be said that the natural frequencies are directly related to the stiffness of the structure. Therefore, a drop in the natural frequencies will indicate a loss of the stiffness. Unfortunately, these changes do not indicate the location of damage. They are insufficient alone to locate the damage.

Table 2. Percentage reduction of natural frequencies due to damage

Case of damage	Damage location (Element No.)	% reduction in natural frequency				
		Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
Case-1	1	0.0133	0.0192	0.0506	0.0613	0.1058
Case-2	8	0.3575	0.4348	0.6684	0.5998	0.3468
Case-3	15	0.6954	0.4904	0.0502	0.0796	0.6112
Case-4	30	0.0034	0.3151	0.0128	0.2755	0.0271
Case-5	30	0.0133	1.1848	0.0502	1.0232	0.1049
Case-6	1,8,15,30	1.0744	2.0918	0.8164	1.7493	1.2126

5.2. Curvature techniques

5.2.1. Number of mode shapes

The damage indices of mode shape curvatures of *Case 4* of damage (10% reduction in E in element No. 30) are plotted in Figs. 2 (a), (b) and (c) for the first, second and fifth mode shapes, respectively. It is shown that the damage indices have clear spikes at the damaged element for each of the first five modes. The values of the damage indices beyond the damaged region have small values, (< 2), which represent the noise (measurements or approximation errors). However, there is a variation in the damage indices values from one mode to another. This depends on the sensitivity of each mode to a specific location of damage. So, the mode shape curvature for each of the first five mode shapes is able to localize

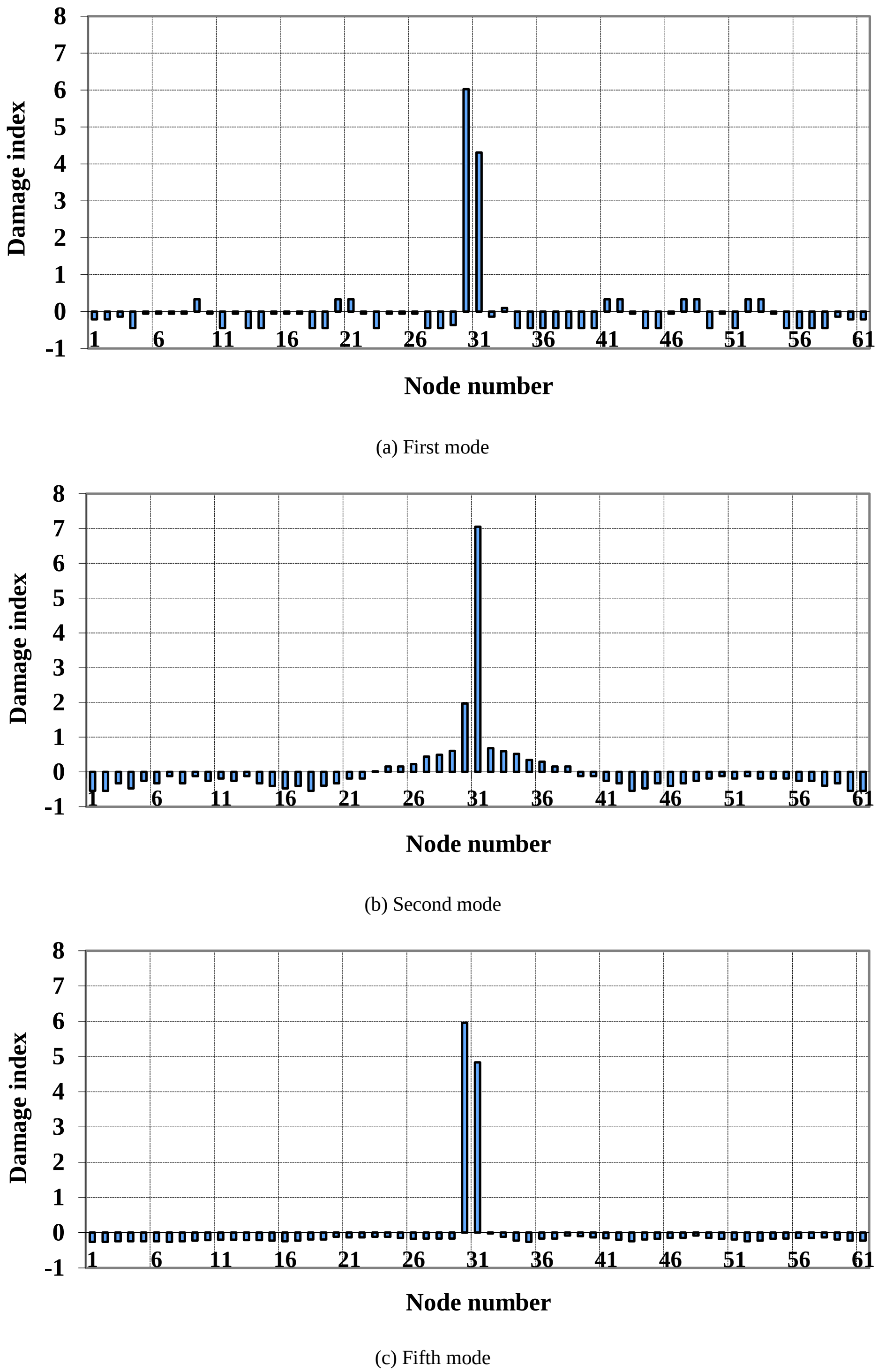
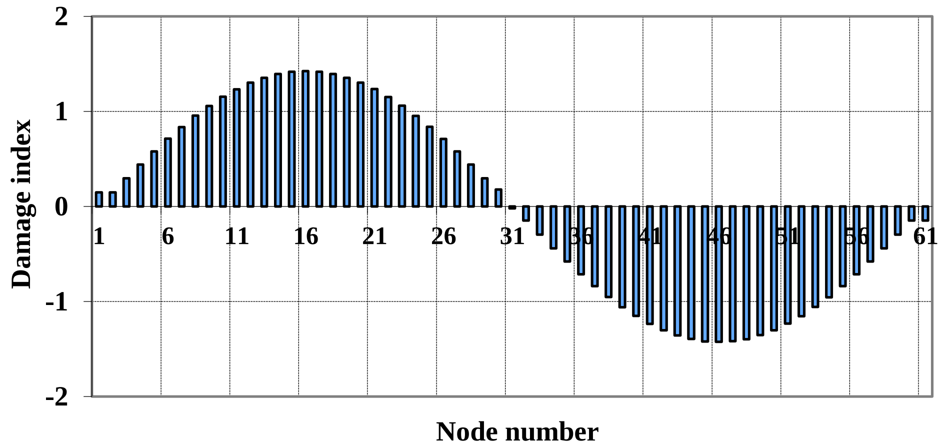
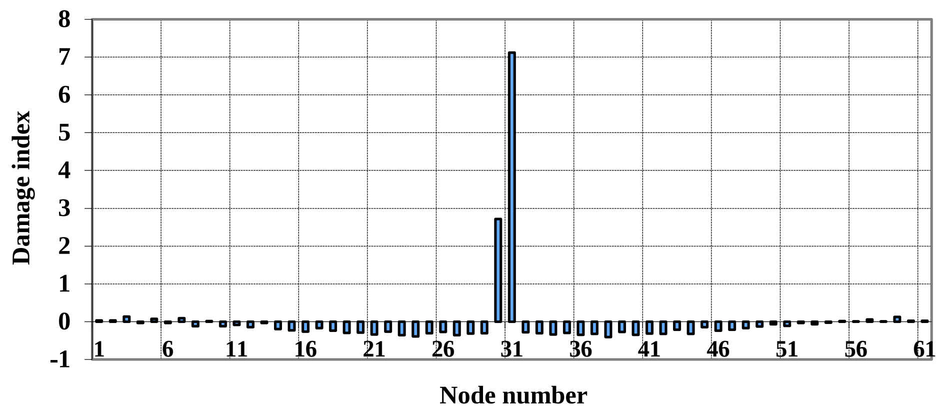


Fig. 2. Damage indices of mode shape curvature of *Case 4* of damage

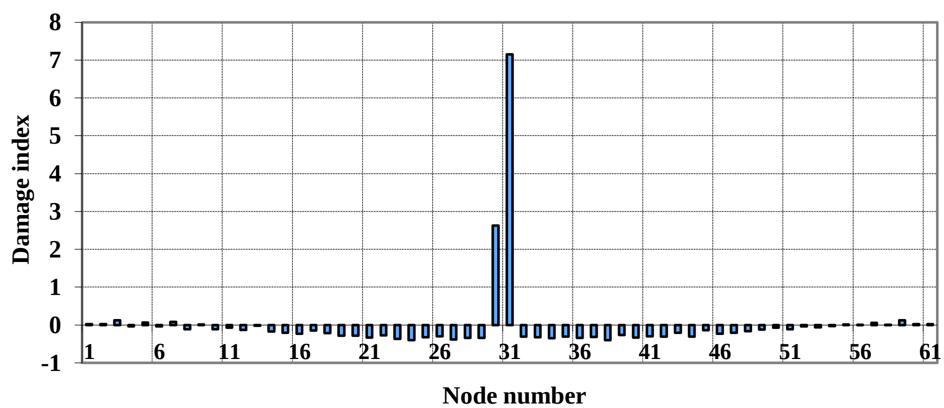
structural damage, even with only the first mode shape. So, in the next section only the first mode shape will be used in the comparison study. It is worth to mention that no need to the natural frequency in calculating the mode shape curvature.



(a) One mode



(b) Two modes



(c) Five modes

Fig. 3. Damage indices of ULS curvature of *Case 4* of damage

As mentioned before, and unlike the mode shape curvature, the ULS curvature technique requires both the natural frequencies and mode shapes. Also, since the ULS depends on the flexibility matrix of the system, so the ULS curvature technique depends upon the number of available modes used in the analysis. Figures 3 (a), (b) and (c) plot the damage indices of ULS curvatures of *Case 4* of damage (10% reduction in *E* in element No. 30) using the first mode, the first two modes and the first five modes, respectively. It is shown that the damage indices of the ULS curvature using only the first mode shape do not give any idea of the damage location. However, the damage indices of ULS curvature using the first two modes and using the first five modes (Figs. 3(b) and (c)) have clear spikes at the damaged element of the beam. Again, the values of the damage indices beyond the damaged region have small values, (< 2), which represent the noise. It is also noticed that Figs. 3(b) and (c) give approximately the same results which imply that the ULS is stable using only the first two mode shapes. Similar results are obtained for *Case 5* of damage (30% reduction in *E* in element No.30). So, in the next section only the first two mode shapes will be used in the comparison study.

5.2.2. One damage location

The damage indices of mode shape curvatures of *Case 1* of damage (30% reduction in *E* in element No. 1) are plotted in Fig. 4 (using only the first mode shape). Also, the damage indices of ULS curvatures of the same case of damage are plotted in Fig. 5 (using the first two mode shapes). Figures 6 and 7 show the same set of results for *Case 3* of damage (30% reduction in *E* in element No. 15). It is shown that the damage indices have clear spikes at the damaged element for both of the two methods. The values of the damage indices beyond the damaged region have small values, (< 2), which represent the noise. However, the ULS curvature method has less noise compared to the mode shape curvature method. This is because the summation of the modal coefficients of more than one mode averages out the random error at each measuring point. Similar results are obtained for *Case 2* of damage. Therefore, the mode shape curvature and the ULS curvature can accurately locate different types of one damage location, i.e., damage due to shear (*Case 1*), damage due to combined shear and bending (*Cases 2, 4 and 5*), and damage due to bending moment (*Case 3*).

5.2.3. Multiple damage locations

The damage indices of mode shape curvatures of Case 6 of damage (30% reduction in E in elements No. 1, 8, 15 and 30) are plotted in Fig. 8 (using only the first mode shape). Also, the damage indices of ULS curvatures of the same case of damage are plotted in Fig. 9 (using the first two mode shapes). It is shown that the damage indices of the two methods of damage detection have clear spikes at the damaged elements of the beam model and have small values beyond the damaged elements. It is also shown that the damage indices of the ULS curvature have less noise compared to those of the mode shape curvature. Indeed, although both of the two methods have clear spikes at the four damage locations, the damage indices of element No. 1 are underestimated (< 2). This can be interpreted by the fact that the damage in element No. 1 is located in a region of zero bending moment and consequently zero curvature. Therefore, the curvatures are less sensitive to that location of damage compared with other damage locations where the curvature has a specific value. Fortunately, most of damage types are related to flexural failures, so the curvature techniques are promising in structural damage identification.

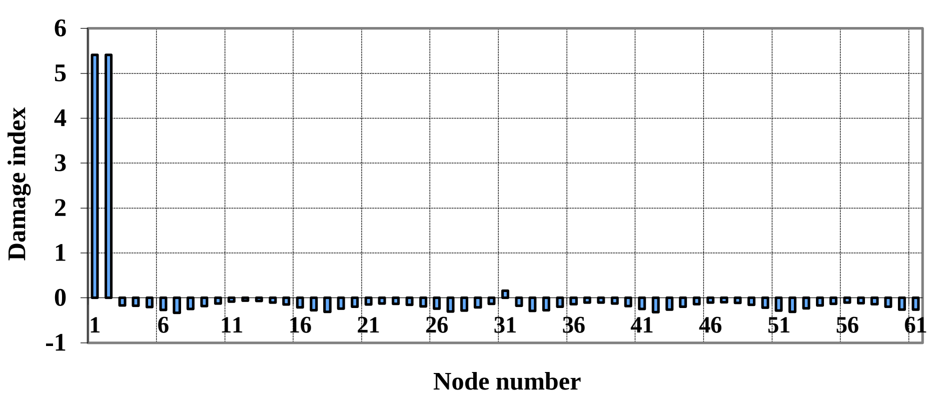


Fig. 4. Damage indices of mode shape curvature of *Case 1* of damage

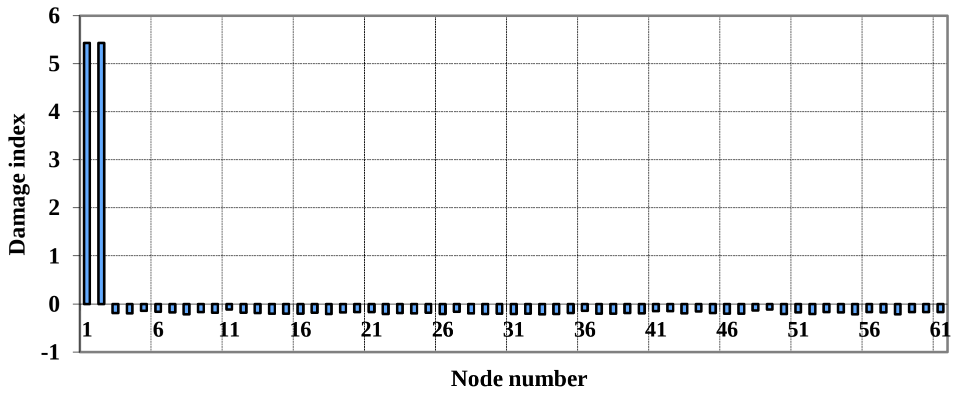


Fig. 5. Damage indices of ULS curvature of *Case 1* of damage

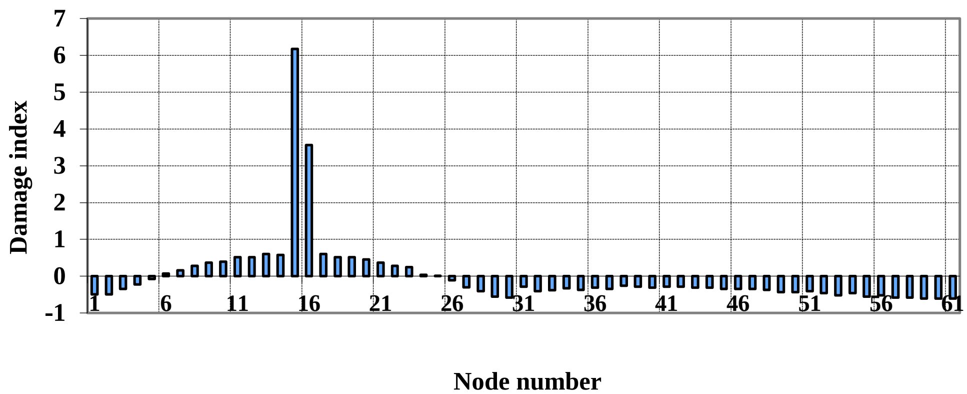


Fig. 6. Damage indices of mode shape curvature of *Case 3* of damage

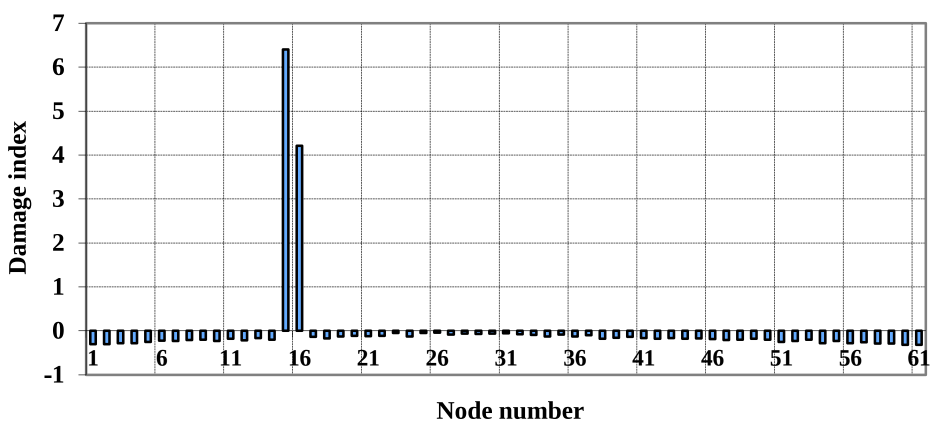


Fig. 7. Damage indices of ULS curvatures of *Case 3* of damage

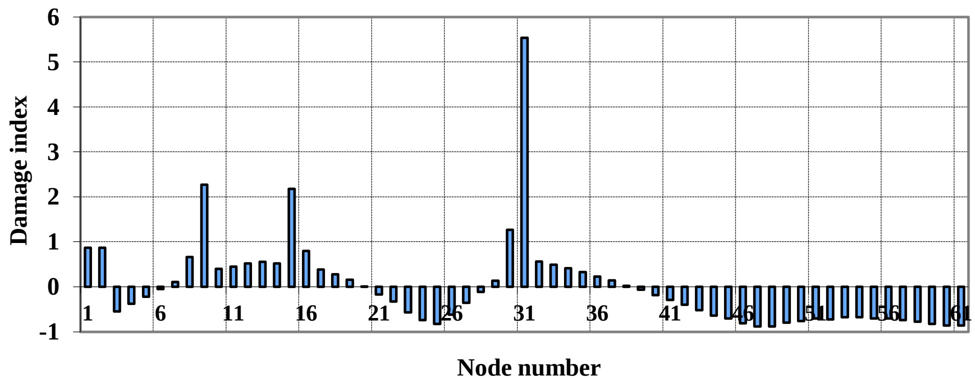


Fig. 8. Damage indices of mode shape curvature of *Case 6* of damage

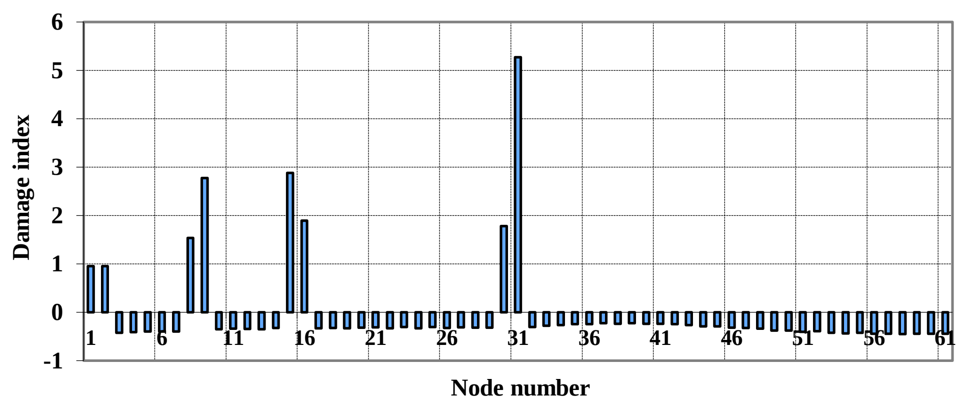


Fig. 9. Damage indices of ULS curvatures of *Case 6* of damage

6. Conclusions

A comparative study of curvature techniques used in structural damage detection, represented here in a numerical finite element model, has been investigated. The results of the two-span continuous steel beam model demonstrate the robustness of the changes in mode shape curvatures and uniform load surface (ULS) curvature as diagnostic parameters in detecting and locating different damage characteristics (one or multiple damage locations with different severities of damage). The robustness in using changes in curvatures is that the two methods require only the lower modes of a structure which can be easily and accurately measured in modal testing. However, the following conclusions are drawn from the above results:

1) Both of the two methods can accurately locate different damage characteristics using the damage localization algorithm.
2) The ULS curvature method requires at least the data of two of the lower modes (natural frequencies and mode shapes) to give an idea about the damage location.
3) The results of the ULS curvature method contain less noise than that of the results of the mode shape curvature method.
4) The mode shape curvature can pinpoint damage locations even with one of the lower mode

shapes of the structure and it does not require the mode frequency.

Indeed, mode shape curvature method has an advantage that the curvature of mode shape can be measured directly without approximation, which will improve the results of damage identification. The details of curvature gauges to measure curvature directly are given by Kovacevic *et al.*, 2006.

References

Doebling, S. W., Farrar, C. R., Prime, M. P., and Shevitz, D. W., 1996. Damage Identification and Health Monitoring of Structural and Mechanical Systems from Changes in Their Vibration Characteristics, *A Literature review*, LA-13070-MS.

Salawu, O. S., 1997. Detection of Structural Damage through Changes in Frequency, *A review, Engineering Structures Journal*, Vol. 19, No. 9, pp. 718-723.

Salawu, O. S. and Williams, C., 1995. Bridge Assessment Using Forced-Vibration Testing, *J. Structural Engineering*, ASCE, Vol. 121, No. 2, pp. 161-173.

Abdo, M. A.-B. and Hori, M., 2001. Structural Damage Detection Using Changes in Mode Shape Derivatives, 3rd *International Summer Symposium*, Japan, August 8, pp. 85-88.

Pandey, A. K., Biswas, M., and Samman, M. M., 1991. Damage Detection from Changes in Curvature Mode Shapes, *J. Sound and Vibration*, Vol. 145, No. 2, pp. 321-332.

Quan, Q. and Weiguo, Z., 1998. Damage Detection of Suspension Bridges, *16th International Modal Analysis Conference (IMAC)*, California, USA, Feb. 2-5, V. 2, pp. 945-951.

Ratcliffe, C. P., 1997. Damage Detection Using a Modified Laplacian Operator on Mode Shape Data, *Journal of Sound and Vibration*, Vol. 204, No. 3, pp. 505-517.

Maeck, J., and De Roeck, G., 1999. Damage Detection on a Prestressed Concrete Bridge and RC Beams using Dynamic System Identification, *Key Engineering Materials*, Vol. 167, pp. 320-327.

Pirner, M., and Urushadze, S., 2001. Localization of Imperfections and Failures of Concrete Structures, *19th International Modal Analysis Conference* (IMAC), Florida, USA, Feb. 5-8, pp. 1370-1373.

Abdo, M. A.-B. and Abdou, M. A.-N., 2005. Reliability of using High-Order Mode Shapes in Structural Damage Detection, *J. Engineering Sciences, Faculty of Engineering, University of Assiut*, Vol. 33 (6), pp. 2051-2068.

Abdo, M. A.-B., 2012. Damage Detection in Plate-Like Structures Using High-Order Mode Shape Derivatives, International Journal of Civil and Structural Engineering, Vol. (2), No. (3), pp. 792-807.

Raghavendrachar, M. and Aktan, A.E., 1992. Flexibility of Multi-Reference Impact Testing for Bridge Diagnostics, *Journal of Structural Engineering, ASCE*, 118, pp. 2186-2203.

Zhao, J., and Dewolf, J. T., 1999. Sensitivity Study for Vibrational Parameters Used in Damage Detection, *Journal of Structural Engineering, ASCE,* Vol. 125, No. 4, pp. 410-416.

Pandey, A. K. and Biswas, M., 1994. Damage Detection in Structures using Changes in Flexibility, *Journal of Sound and Vibration*, Vol. 169, No. 1, pp. 3-17.

Pandey, A. K. and Biswas, M., 1995. Experimental Verification of Flexibility Difference Method for Locating Damage in Structures, *Journal of Sound and Vibration*, Vol. 184, No. 2, pp. 311-328.

Lu, Q., Ren, G. and Zhao, Y., 2002. Multiple Damage Location with Flexibility Curvature and Relative Frequency Change for Beam Structures, *Journal of Sound and Vibration*, Vol. 253, No. 5, pp. 1101-1114.

Zhang, Z. and Aktan, A. E., 1995. The Damage Indices for the Constructed Facilities, *13th International Modal Analysis Conference* (IMAC), Society for Experimental Mechanics, USA, Vol. 2, pp. 1520-1529.

Wu, D. and Law, S. S., 2004. Damage Localization in Plate Structures from Uniform Load Surface Curvature, Journal of Sound and Vibration, 276 (1-2), pp. 227- 244.

Stubbs, N., Kim, J.-T., and Farrar, C. R., 1995. Field Verification of a Nondestructive Damage Localization and Severity Estimation Algorithm, *13th International Modal Analysis Conference*, (IMAC), Society for Experimental Mechanics, USA, pp. 210-218.

MARC, 2010-a. Analysis Research Corporation, *Volumes; A, B, and C*, Version 2010.

MARC, 2010-b. Analysis Research Corporation, *Mentat User's Guide*, Version 2010.

Kovacevic, M., Djordjevich, A. and Nikezic, D., 2006. An Analytical Approach and Optimization of Curvature Gauge, J. Physics: Conference Series 48, pp. 850-858