Effect of Radiation and Double Dispersion on Mixed Convection Heat and Mass Transfer in Non-Darcy Porous Medium

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Abstract

Similarity solution for the problem of hydrodynamic dispersion and radiation in non- Darcy mixed convection heat and mass transfer from vertical surface embedded in porous media is presented. The Forchheimer extension is considered in the governing equations. The heat and mass transfer in the boundary layer region for aiding and opposing buoyancies in both flows has been analyzed. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. The dimensionless velocity, temperature and concentration fields in the non-Darcy porous media are governed by complex interactions among the diffusion rate, buoyancy ratio and radiation parameter in addition to the flow driving parameter. Numerical results for details of the dimensionless velocity, temperature and concentration which shown on graphs and tables have been presented and compared against previously published work on special cases of the problem and found to be in excellent agreement. The combined effects of radiation, thermal dispersion and solutal diffusivity, for the non-Darcy porous medium, on the dimensionless velocity and heat and mass coefficients are discussed.

Keywords: Radiation ; Mixed Convection; Double Dispersion ; Porous Medium; Non-Darcy; Boundary Layer ; Heat and Mass Transfer

1. Introduction

Thermal and solutal transport by fluid flowing through a porous matrix is a phenomenon of great interest from both the theory and application point of view. The flow phenomenon is relatively complex rather than that of the pure thermal convection process. Heat and mass transfer processes in porous media are often encountered in the study of dynamics hot and salty springs of a sea, and in the chemical industry, in reservoir engineering about thermal recovery process. and other pollutants, grain storage, evaporation cooling, and solidification are the few other application areas where the combined thermo-solutal mixed convection in porous media are observed.

Combined heat and mass transfer by free convection under boundary layer approximations has been studied by Bejan and Khair [1] and Murthy and Singh [2]. Mixed convection boundary layer flow on a surface in a saturated porous medium was studied by Merkin [3]. Coupled heat and mass transfer by mixed convection in Darcian fluid-saturated porous medium has been analyzed by Lai [4]. Thermal dispersion effects have been studied by Lai and Kulacki [5], Amiri and Vafai [6] and Murthy and Singh [7]. Coupled heat and mass transfer phenomenon in non-Darcy flows are studied by Karimi-Fard et al. [8] and Murthy and Singh [2]. The effect of solutal and thermal dispersion effects in homogeneous and isotropic Darcian porous media has been analyzed by Dagan [9]. A systematic derivation of the governing equations with various types of approximations used in applications has been presented. Using scale analysis arguments, Telles, and Trevisan [10] analyzed the double dispersion phenomenon in a free convection boundary layer adjacent to a vertical wall in a Darcian fluid-saturated porous medium. Murthy and Singh [11] studied the convective heat transfer in non-Darcy porous media. The effect of double dispersion on mixed convection heat and mass transfer in non-Darcy porous medium has been analyzed under boundary layer approximations using the similarity solution technique by Murthy [12]. Mansour and El-Amin [13] studied the thermal dispersion effects on non-Darcy axisymmetric free convection in a saturated porous medium. Double dispersion effects on natural convection heat and mass transfer in non-Darcy porous medium studied by El-Amin [14].

and Takhar [15] studied the effects of thermal radiation on mixed convection along a vertical plate subjected to uniform surface temperature. The problem of steady two-dimensional free convection flow through a very porous medium bounded by a vertical infinite porous plate by the presence of thermal radiation was considered by Raptis [16]. The problem of thermal dispersion-radiation effects on non-Darcy natural convection in a fluid saturated porous medium studied by Mohammadien and El-Amin [17]. Thermal radiation effect on non-Darcy natural convection with lateral mass transfer investigated by El-Hakiem and El-Amin [18]. El-Hakiem [19] studied radiative effects on non-Darcy natural convection from a heated vertical plate in saturated porous media with mass transfer for non-Newtonian fluid. Heat and mass transfer by non-Darcy free convection from a vertical cylinder embedded in porous media with a temperature dependent viscosity investigated by Chamkha, et al. [20]. Effect of rotation on thermal convection in an anisotropic porous medium with temperature dependent viscosity was studied by Vanishree [21]. El-Hakiem et al. [22] studied Natural convection boundary layer of non-Newtonian fluid about a permeable vertical Cone embedded in porous medium saturated with a nanofluid. Chamkha, A.J., et.al. [23] studied Coupled heat and mass transfer by MHD free convection flow along a vertical plate with streamwise temperature and species concentration variations. Nield and Bejan [24] explained about convective heat transfer in porous medium in their book. Kairi and Murthy [25] investigated the effect of double dispersion on mixed convection heat and mass transfer in a non-Newtonian fluid-saturated non-Darcy porous medium. Srinivasacharya, D., et. al. [26] studied mixed convection heat and mass transfer along a vertical plate embedded in a power-law fluid saturated Darcy porous medium with chemical reaction and radiation effects.

The present work, investigates the effects of double dispersion-radiation on mixed convection heat and mass transfer in non-Darcy porous medium. The Forchheimer flow model is considered and the porous medium porosity is assumed to be low so that the boundary effects in the medium may be neglected. The heat and mass transfer in the boundary layer region has analyzed for aiding and opposing buoyancies for both aiding and opposing flows. The dimensionless velocity, temperature and concentration fields in non-Darcy porous media are observed to be governed by complex interactions among the diffusions rate Le, buoyancy ratio N, Pe_γ and Pe_ξ the dispersion thermal and solutal diffusivity parameters respectively and radiation parameter R. The Rosseland approximation is used to describe the radiative heat flux in the energy equation.

2. Analysis

Mixed convection heat and mass transfer from the impermeable vertical flat wall in a fluid-saturated porous medium is considered for the study. The x-axis is taken along the plate and the y-coordinate normal to it. The wall is maintained at constant temperature and concentration, T_w and C_w respectively, and these values are assumed to be greater than the ambient temperature and concentration T_∞ and C_∞ respectively. The gravitational acceleration g is in a direction opposite to x-direction. The radiative heat flux in the x-direction is consider negligible in comparison with that in the y-direction. The governing equations for the boundary layer flow, heat and mass transfer from the wall $y=0$ into the fluid-saturated porous medium $x \geq 0$ and $y>0$ (after making use of the Boussinesq approximation).

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial y}+\frac{c\sqrt{K}}{\nu}\frac{\partial u^2}{\partial y}=\left(\frac{Kg\beta_T}{\nu}\right)\frac{\partial T}{\partial y}+\left(\frac{Kg\beta_C}{\nu}\right)\frac{\partial C}{\partial y} \quad ,(2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha_{ff}\frac{\partial T}{\partial y}\right)-\frac{\alpha}{k}\frac{\partial q^r}{\partial y}, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{\partial}{\partial y}\left(D_C\frac{\partial C}{\partial y}\right), \tag{4}$$

The boundary conditions are:

$$y=0: \quad v=0, \quad T=T_w, \quad C=C_w$$

$$y\to\infty: \quad u=u_\infty, \quad T=T_\infty, \quad C=C_\infty \tag{5}$$

Here x and y are the Cartesian coordinates, u and v are the averaged velocity components in x and y-directions respectively; T is the temperature; C is the concentration; ν is the kinematic viscosity of the fluid; β_T is the coefficient of thermal expansion; β_C is the coefficient of solutal expansion; K the permeability of the porous medium; k the thermal conductivity; c an empirical constant; g the acceleration due to gravity; α the equivalent thermal diffusivity of the porous medium; α_{ff} and D_C are the effective thermal and solutal diffusivities, respectively. Telles and Trevisan (1993) investigate $\alpha_{ff} = \alpha + \gamma du$, $D_C = D + \xi du$, whereas γdu and ξdu represent dispersion thermal and solutal diffusivities, respectively. The quantity q^r represents the radiative heat flux in the y-direction. The radiative heat flux term is simplified by using the Rosseland approximation as:

$$q^r = -\frac{4\sigma}{3k_0}\frac{\partial T^4}{\partial y}, \tag{6}$$

where σ and k_0 are the Stefan-Boltzman constant and the mean absorption coefficient, respectively.

Proceeding with the analysis, we define the following transformations:

$$\eta = \left(\frac{y}{x}\right)Pe_x^{1/2}, \qquad f(\eta) = \frac{\psi}{\alpha Pe_x^{1/2}},$$

$$\theta(\eta) = \frac{(T - T_\infty)}{(T_w - T_\infty)}, \ \phi(\eta) = \frac{(C - C_\infty)}{(C_w - C_\infty)} \tag{7}$$

Substituting the expressions in (7) into equations (2), (3) and (4), the transformed governing equations may be written as:

$$f'' + 2F_0 Pef'f'' = \pm\frac{Ra}{Pe}[\theta' + N\phi'] \tag{8}$$

$$\theta'' + \frac{1}{2}f\,\theta' + Pe_\gamma[f'\theta'' + f''\theta'] + \frac{4R[(\theta + C_T)^3\theta']'}{3} = 0 \tag{9}$$

$$\phi'' + \frac{1}{2}Lef\phi' + LePe_\xi[f'\phi'' + f''\phi'] = 0 \tag{10}$$

The boundary conditions (5) transform into

$$\eta = 0 : f = 0, \theta = 1, \phi = 1,$$

$$\eta \to \infty : f' = 1, \theta = 0, \phi = 0 \tag{11}$$

The important parameters involved in the present study are the radiation parameter $R = \frac{4\sigma}{3k_0 k}(T_w - T_\infty)^3$, temperature difference $C_T = \frac{T_\infty}{T_w - T_\infty}$, the local Peclet number $Pe_x = \frac{U_\infty x}{\alpha}$ the local Darcy-Rayleigh number $Ra_x = \frac{gK\beta_T\theta_w x}{\alpha\nu}$, which is defined with reference to the temperature difference alone, $Pe = \frac{U_\infty d}{\alpha}$ and $Ra = \frac{gK\beta_T\theta_w d}{\alpha\nu}$, are the pore diameter-dependent Peclet and

Rayleigh number, respectively. The inertial parameter is $F_0 Pe = \frac{c\sqrt{K}U_\infty}{\nu}$ (in the present study $F_0 Pe = 1.0$) the buoyancy ratio is $N = \frac{\beta_C \phi_w}{\beta_T \theta_w}$ and the diffusivity ratio is $Le = \alpha / D$. The Lewis number is nothing but the ratio of the Schmidt number ν / D and Prandtl number ν / α. The flow governing parameter is Ra / Pe and is independent of x. $Ra / Pe = 0$ represents the forced convection flow. The flow asymptotically reaches the free convection flow limit as this parameter tends to ∞. Pe_γ and Pe_ξ represent thermal and solutal dispersion parameters, respectively, and are defined here as $Pe_\gamma = \frac{\gamma U_\infty d}{\alpha}$ and $Pe_\xi = \frac{\xi U_\infty d}{\alpha}$. In the present investigation, we consider the thermal and solutal dispersion parameters Pe_γ and Pe_ξ with γ and ξ included in the parameters. In Eq. (8) the positive and negative signs represent aiding and opposing flows, respectively, $N > 0$ indicates the aiding buoyancy and $N < 0$ indicates the opposing buoyancy.

The heat and mass transfer coefficients, in terms of the Nusselt and Sherwood numbers in the presence of radiation and thermal and solutal dispersion diffusivities, can be written as

$$q_w = -k_c \frac{\partial T}{\partial y} - \frac{4\sigma}{3k_0} \frac{\partial T^4}{\partial y}\bigg|_{y=0},$$

q_w the effect heat transfer coefficient; $k_c = k + k_d$ where k_c effective thermal conductivity; k_d the dispersion thermal conductivity

$$j_w = -D_c \frac{\partial C}{\partial y}\bigg|_{y=0},$$

j_w the convective mass transfer coefficient; D_c mass diffusivity

$$\frac{Nu}{Pe_x^{1/2}} = \frac{q_w}{T_w - T_\infty} \frac{x}{k_e} = -[1 + Pe_\gamma f'(0) + \frac{4}{3} R (C_T + \theta(0))^3] \theta'(0) \tag{12}$$

$$\frac{Sh}{Pe_x^{1/2}} = \frac{j_w x}{D_c (C_w - C_\infty)} = -[1 + Pe_\xi f'(0)] \varphi'(0), \tag{13}$$

when $R = 0$, $Pe_\gamma = Pe_\xi = 0$, Eqs. (12)-(13) become $\frac{Nu}{Pe_x^{1/2}} = -\theta'(0)$ and $\frac{Sh}{Pe_x^{1/2}} = -\phi'(0)$. For $Pe_\gamma = Pe_\xi = 0$, $R \neq 0$ then

$$\frac{Nu}{Pe_x^{1/2}} = -[1 + \frac{4}{3} R(C_T + \theta(0))^3] \theta'(0)$$

3. Results and Discussion

The resulting ordinary differential Eqns. (8)-(10) are integrated by giving appropriate initial guess values for $f'(0)$, $\theta'(0)$ and $\phi'(0)$ to match the values with the corresponding boundary conditions at $f'(\infty)$, $\theta(\infty)$ and $\phi(\infty)$ respectively.

NAG software (D02HAFE routine) is used for integrating the corresponding first-order system of equations and shooting and matching the initial and boundary conditions. The step size $\Delta\eta = 0.05$ is used while obtaining the numerical solution with $\eta_{max} = 12$ and five –decimal accuracy as the criterion for convergence. Extensive calculations have been performed to obtain the flow, temperature and concentration fields for a wide range of parameter $0 \le Ra/Pe \le 100$, $0 \le R \le 1.0$, $F_0Pe = 1.0$, $N = -0.5$, 1.0, $0.1 \le Le \le 100$, $C_T = 0.01$, $0 \le Pe_\gamma \le 5$, and $0 \le Pe_\xi \le 5$. With $R = 0$, $F_0Pe = 0$, (Darcian case), $Pe_\gamma = 0$ and $Pe_\xi = 0$ the present problem reduces to heat and mass transfer by Darcian mixed convection in porous media analyzed by Lia [4], and for $F_0Pe = 1.0$ with variation of another parameter compared with Murthy (2000).

Aiding flow: when buoyancy is aiding the flow, for $N > 0$ (aiding buoyancy case) the tangential velocity evolves from nonzero wall velocity to uniform freestream velocity for all values of $N > 0$. The vertical component of velocity attains negative values near the wall and well inside the boundary layer.

The wall temperature gradient values for $F_0Pe = 1.0$ for two values of $N = -0.5$ and $N = 1.0$ are presented in Tables (II-III). The value of $f'(0)$ is independent on R. From these tables, it is clear that $f'(0)$ depends on the buoyancy ratio N. The variation of the heat transfer coefficient with Ra/Pe for nonzero values of Pe_γ is studied for a wide range of values of Le. The effect of thermal dispersion on the heat transfer is studied keeping $Pe_\xi = 0$. Consistent with the results presented in Lai and Kulacki (1991b), the value of $-\theta'(0)$ decreases as the thermal dispersion coefficient Pe_γ increases.

Table 1: Values for $f'(0)$ and $-\theta(0)$ for varying Ra/Pe, Pe_γ with $Pe_\xi = 0$, $Le = 1$

		Murthy (2000)			Present work			
$\frac{Ra}{Pe}$	$f'(0)$	$Pe_\gamma=0$	$Pe_\gamma=1$	$Pe_\gamma=5$	$f'(0)$	$Pe_\gamma=0$	$Pe_\gamma=1$	$Pe_\gamma=5$
0.0	1.0	0.56421	0.39895	0.23044	1.0	0.56419	0.39894	0.23045
1.0	1.15834	0.59224	0.40279	0.23068	1.15831	0.59223	0.40277	0.23066
5.0	1.67946	0.67933	0.41884	0.23564	1.67944	0.67930	0.41881	0.23559
10	2.19261	0.75803	0.43484	0.24077	2.19258	0.75804	0.43481	0.24072
20	3.1000	0.87062	0.45627	0.24688	3.0	0.87055	0.45622	0.24678
50	4.72062	1.07684	0.48784	0.25447	4.72014	1.07681	0.48782	0.25442

Table 2: **Variation of** $-\theta'(0)$ **for varying of** R, Ra/Pe, Pe_γ **and** Le **with** $Pe_\xi = 0$, $N = -0.5$

			$Le = 1.0$			$Le = 10$		
R	$\frac{Ra}{Pe}$	$f'(0)$	$Pe_\gamma = 0.0$	$Pe_\gamma = 1.0$	$Pe_\gamma = 5.0$	$Pe_\gamma = 0.0$	$Pe_\gamma = 1.0$	$Pe_\gamma = 5.0$
	0.0	1.0	0.56419	0.39894	0.23045	0.56419	0.39894	0.23045
0.0	1.0	1.15831	0.59223	0.40277	0.23066	0.60540	0.41525	0.23699
	5.0	1.67944	0.67930	0.41881	0.23559	0.72436	0.43500	0.24041
	10	2.19258	0.75804	0.43483	0.24072	0.82473	0.45104	0.26959
	20	3.0	0.87055	0.45622	0.24678	0.96171	0.47823	0.30644
	50	4.72014	1.07681	0.48782	0.25442	1.20301	0.50729	0.32186
	0.0	1.0	0.37535	0.31573	0.21125	0.37535	0.31573	0.21125
	1.0	1.15831	0.39641	0.32513	0.21387	0.40513	0.33464	0.21952
	5.0	1.67944	0.46052	0.35362	0.22330	0.48921	0.38192	0.23688
	10	2.19258	0.51733	0.37732	0.23079	0.55896	0.41443	0.25596
	20	3.0	0.59736	0.40693	0.23907	0.65332	0.45012	0.27931
0.5	50	4.72014	0.74209	0.44934	0.24913	0.81843	0.48891	0.30564
	0.0	1.0	0.29251	0.26541	0.19552	0.29251	0.26541	0.19552
	1.0	1.15831	0.31030	0.27666	0.19979	0.31701	0.28432	0.20490
	5.0	1.67944	0.36366	0.30942	0.21250	0.38517	0.33254	0.22500
	10	2.19258	0.41031	0.33615	0.22283	0.43379	0.35999	0.24186
	20	3.0	0.47539	0.36943	0.23194	0.51853	0.40153	0.27311
1.0	50	4.72014	0.59211	0.41781	0.24411	0.65721	0.45391	0.29655

Also for large Pe_γ, in a very small region near the wall, temperature gradient is greatly increased and as a result heat transfer is greatly enhanced due to thermal dispersion. The value of $-\theta'(0)$ decreases as the radiation parameter R increases. The value $-\theta'(0)$ increases with increase Ra/Pe, and the value of Le enhances of $-\theta'(0)$ when $N = -0.5$, but, it reduce when $N = 1.0$.

The effect of radiation and solutal dispersion on the mass transfer coefficient has been analyzed keeping $Pe_\gamma = 0$. The values of $-\phi'(0)$ have been tabulated for $F_0 Pe = 1.0$, $N = -0.5$ and $N = 1.0$ in Tables (VI-V). Analogous to the pure thermal convection process, the value of $-\phi'(0)$ decreases with increasing values of Pe_ξ. But the increase value of radiation parameter R enhance $-\phi'(0)$ with fixed

Table 3: Variation of $-\theta'(0)$ for varying of R, Ra/Pe, Pe_γ and Le with $Pe_\xi = 0$, $N = 1.0$.

			$Le = 1.0$			$Le = 10$		
R	$\frac{Ra}{Pe}$	$f'(0)$	$Pe_\gamma = 0$	$Pe_\gamma = 1$	$Pe_\gamma = 5$	$Pe_\gamma = 0$	$Pe_\gamma = 1$	$Pe_\gamma = 5$
0.0	0.0	1.0	0.56419	0.39894	0.23045	0.56419	0.39894	0.23045
	1.0	1.56155	0.66027	0.38933	0.19845	0.63766	0.37046	0.18999
	5.0	3.00000	0.87055	0.37742	0.16944	0.80833	0.34066	0.15702
	10	4.21699	1.02028	0.37172	0.16080	0.86751	0.33579	0.14895
	20	6.00000	1.20974	0.36603	0.15505	0.99757	0.32437	0.14366
	50	9.61187	1.52680	0.35883	0.15054	1.11689	0.31386	0.13973
0.5	0.0	1.0	0.37535	0.31573	0.21125	0.37535	0.31573	0.21125
	1.0	1.56155	0.43978	0.32316	0.18713	0.42471	0.30825	0.17940
	5.0	3.00000	0.53500	0.33388	0.16419	0.53958	0.30297	0.15247
	10	4.21699	0.68033	0.33807	0.15724	0.64185	0.28443	0.15016
	20	6.00000	0.80676	0.34092	0.15264	0.78793	0.26977	0.14879
	50	9.61187	1.01826	0.34241	0.14909	0.99985	0.25318	0.14553
1.0	0.0	1.0	0.29251	0.26541	0.19552	0.29251	0.26541	0.19552
	1.0	1.56155	0.34276	0.27935	0.17729	0.33107	0.26706	0.17019
	5.0	3.00000	0.45234	0.30110	0.15934	0.42112	0.26706	0.14824
	10	4.21699	0.53026	0.31120	0.15388	0.40053	0.27455	0.13722
	20	6.00000	0.62880	0.31979	0.15033	0.38839	0.25311	0.13019
	50	9.61187	0.79365	0.32782	0.14768	0.36875	0.23379	0.12731

parameter R enhance $-\phi'(0)$ with fixed the other parameters. Interestingly, at large Pe_ξ, for large values of Ra/Pe, in a relatively large region (larger than that observed for thermal gradients) near the wall, the concentration gradient is greatly increased. But, against this expectation peculiar behavior in the mass transfer coefficient is observed (see Murthy (2000)). The imbalance between the Lewis number and buoyancy parameter influence more against the enhancement of the mass transfer results.

For $Pe_\xi = 0$, the value of $-\phi'(0)$ increases with increasing values of Ra/Pe for all values of Le and N. For large Pe_ξ, the value of $-\phi'(0)$ decreases rapidly to near zero values with increasing Ra/Pe.

The variation of $-\frac{Nu}{Pe_x^{1/2}}$ with R and Ra/Pe in the opposing flow is presented in Table (VI) with $F_0 Pe = 1.0$ for different value of ($Pe_\gamma = 0, 1, 5$) and ($N = -0.5$, 1.0). Thermal dispersion and radiation

Table 4: Variation of $-\phi'(0)$ for varying of R, Ra/Pe, Pe_ξ and Le with $Pe_\gamma = 0$, $N = -0.5$

			$Le = 1.0$			$Le = 10$		
R	$\frac{Ra}{Pe}$	$f'(0)$	$Pe_\xi = 0$	$Pe_\xi = 1$	$Pe_\xi = 5$	$Pe_\xi = 0$	$Pe_\xi = 1$	$Pe_\xi = 5$
0.0	0.0	1.0	0.56419	0.39894	0.23045	1.78412	0.53793	0.24987
	1.0	1.15831	0.59223	0.38837	0.20630	1.93287	0.51217	0.22082
	5.0	1.67944	0.67930	0.35547	0.14588	2.35336	0.44638	0.15966
	10	2.19258	0.75804	0.32543	0.10350	3.21597	0.40371	0.15280
	20	3.0	0.87055	0.28223	0.06659	3.97841	0.38673	0.14321
	50	4.72014	1.07681	0.21355	0.04841	5.35379	0.31888	0.13857
0.5	0.0	1.0	0.56419	0.39894	0.23045	1.78412	0.53793	0.24987
	1.0	1.15831	0.59663	0.39267	0.20859	1.94121	0.51941	0.22355
	5.0	1.67944	0.69502	0.36934	0.15210	2.37802	0.46650	0.15978
	10	2.19258	0.78190	0.34494	0.11098	3.86879	0.41784	0.14591
	20	3.0	0.90393	0.30698	0.07359	4.68315	0.35397	0.13997
	50	4.72014	1.12404	0.24099	0.05219	6.11943	0.31756	0.13011
1.0	0.0	1.0	0.56419	0.39894	0.23045	1.78412	0.53793	0.24987
	1.0	1.15831	0.59986	0.39605	0.21049	1.94586	0.52485	0.22580
	5.0	1.67944	0.70629	0.38034	0.15742	2.39143	0.48196	0.16572
	10	2.19258	0.79877	0.36061	0.11756	4.33829	0.43527	0.15175
	20	3.0	0.92723	0.32724	0.07983	5.21537	0.40074	0.14462
	50	4.72014	1.19315	0.26425	0.05579	6.97311	0.34769	0.13989

enhances the heat transfer rate in both $N = -0.5$, 1.0 for all values of $Le > 0$. Interestingly, for fixed Le, and nonzero Pe_γ, there exists one critical value of Ra/Pe before which the $-\frac{Nu}{Pe_x^{1/2}}$ for $N = 1.0$ is more than that for $N = -0.5$ and after which its reverse is seen. The $-\frac{Sh}{Pe_x^{1/2}}$ is presented against Ra/Pe for $F_0 Pe = 1.0$ and for six combinations of Pe_ξ and N in Table (VII).

Figure 1 illustrate velocity profiles as a function of the similarity variable η for case aiding buoyancy. Fig. 1 shows the velocity distribution for varying values of radiation parameter R for non-Darcy case ($F_0 Pe = 1.0$), $Pe_\gamma = Pe_\xi = 0$ (aiding flow) with $Le = 1.0$, $N = -0.5$. From this figure we, observe that the velocity profiles increases with increase of parameter radiation. Also, we observe that the increases of Ra/Pe enhances the velocity at fixed the other parameters.

Table 5: Variation of $-\phi'(0)$ for varying of R, Ra/Pe, Pe_ξ and Le with $Pe_\gamma = 0$, $N = 1.0$

			$Le = 1.0$			$Le = 10$		
R	$\frac{Ra}{Pe}$	$f'(0)$	$Pe_\xi = 0$	$Pe_\xi = 1$	$Pe_\xi = 5$	$Pe_\xi = 0$	$Pe_\xi = 1$	$Pe_\xi = 5$
0.0	0.0	1.0	0.56419	0.39894	0.23045	1.78412	0.53793	0.24987
	1.0	1.56155	0.66027	0.38933	0.19845	2.13811	0.47766	0.20788
	5.0	3.00000	0.87055	0.37742	0.16944	2.88656	0.42210	0.17279
	10	4.21699	1.02028	0.37172	0.16080	3.40615	0.40571	0.16533
	20	6.00000	1.20974	0.36603	0.15505	4.05482	0.38722	0.15669
	50	9.61187	1.52680	0.35883	0.15054	5.13491	0.36101	0.14991
0.5	0.0	1.0	0.56519	0.39894	0.23045	1.78412	0.53793	0.24987
	1.0	1.56155	0.66373	0.39235	0.19982	2.14484	0.48244	0.20947
	5.0	3.00000	0.87932	0.38304	0.17134	2.90218	0.42976	0.17485
	10	4.21699	1.03186	0.37778	0.16261	3.46593	0.41079	0.16797
	20	6.00000	1.22435	0.37206	0.15668	4.11205	0.39998	0.15825
	50	9.61187	1.54583	0.36436	0.15192	5.13585	0.36979	0.15011
1.0	0.0	1.0	0.56419	0.39894	0.23045	1.78412	0.53793	0.24987
	1.0	1.56155	0.66632	0.39477	0.20098	2.14861	0.48615	0.21080
	5.0	3.00000	0.88599	0.38772	0.17297	2.91083	0.43604	0.17662
	10	4.21699	1.04074	0.38289	0.16417	3.49215	0.41537	0.16996
	20	6.00000	1.23558	0.37720	0.15809	4.17499	0.40192	0.16007
	50	9.61187	1.56049	0.36909	0.15310	5.24193	0.38129	0.15215

Table 6: Variation of $-\frac{Nu}{Pe_x^{1/2}}$ for varying of R, N, Le, Ra/Pe, Pe_γ with $Pe_\xi = 0$, $F_0 Pe = 1$

			$Le = 1.0$			$Le = 10.0$		
N	R	$\frac{Ra}{Pe}$	$Pe_\gamma = 0$	$Pe_\gamma = 1$	$Pe_\gamma = 5$	$Pe_\gamma = 0$	$Pe_\gamma = 1$	$Pe_\gamma = 5$
-0.5	0.0	0.1	0.56118	0.79052	1.36380	0.55964	0.78747	1.35872
		1.0	0.53189	0.72213	1.19033	0.51409	0.68810	1.13100
-0.5	0.5	0.1	0.62932	0.84034	1.39331	0.62758	0.83724	1.38830
		1.0	0.59167	0.76552	1.21560	0.57084	0.73006	1.15643
-0.5	1.0	0.1	0.68976	0.88688	1.42212	0.68784	0.88372	1.41709
		1.0	0.64454	0.80596	1.24027	0.62109	0.76939	1.18136
1.0	0.0	0.1	0.55187	0.77391	1.33878	0.55500	0.78007	1.34914
		1.0	0.37342	0.47189	0.76294	0.42625	0.55561	0.90153
1.0	0.5	0.1	0.61920	0.82401	1.36885	0.62278	0.83030	1.37913
		1.0	0.41504	0.51168	0.80281	0.47679	0.60257	0.95278
1.0	1.0	0.1	0.67901	0.87074	1.39816	0.68295	0.87712	1.40829
		1.0	0.44783	0.53173	.82759	0.52334	0.64483	0.99189

Table 7: Variation of $-\frac{Sh}{Pe_x^{1/2}}$ for varying of R, N, Le, Ra/Pe, Pe_ξ with $Pe_\gamma = 0$, $F_0 Pe = 1$

			$Le = 1.0$			$Le = 10.0$		
N	R	$\frac{Ra}{Pe}$	$Pe_\xi = 0.0$	$Pe_\xi = 1.0$	$Pe_\xi = 5.0$	$Pe_\xi = 0.0$	$Pe_\xi = 1.0$	$Pe_\xi = 5.0$
-0.5	0.0	0.1	0.56118	0.79346	1.37995	1.76759	1.07270	1.49863
		1.0	0.53189	0.74944	1.34570	1.60035	1.04181	1.48834
-0.5	0.5	0.1	0.56067	0.79244	1.37824	1.76654	1.07085	1.49650
		1.0	0.52621	0.73830	1.32616	1.58674	1.01985	1.46344
-0.5	1.0	0.1	0.56030	0.79165	1.37682	1.76594	1.06948	1.49479
		1.0	0.52193	0.72963	1.31021	1.57878	1.00364	1.44328
1.0	0.0	0.1	0.55187	0.77391	1.33878	1.73797	1.06233	1.46203
		1.0	0.37342	0.47189	0.76294	1.04066	2.53051	4.92592
1.0	0.5	0.1	0.55135	0.77287	1.33697	1.73687	1.06041	1.45983
		1.0	0.36167	0.45430	0.73287	1.00991	2.42579	3.54993
1.0	1.0	0.1	0.55096	0.77206	1.33550	1.73626	1.05898	1.45802
		1.0	0.35323	0.44155	0.71066	0.99160	2.35268	3.48426

The temperature profile for the case aiding buoyancy is presented in Fig. 2. It is evident from this figure the radiation parameter R enhances the temperature profiles. It can be seen that as the buoyancy parameter N increases, the temperature profiles decreases. Also, this figure clearly indicates the favorable influence of the thermal dispersion on the temperature profiles. The temperature profiles θ as a function of η increases with increase of thermal dispersion Pe_γ for $N = -0.5$, $N = 1.0$.

The effect of radiation parameter R, solutal dispersion Pe_ξ and buoyancy parameter N on concentration profiles is plotted in Fig. 3. From this figure, we observe the concentration profiles for the case aiding buoyancy increases with increase the solutal dispersion Pe_ξ, also, it increase with radiation parameter increase. It is noteworthy, from Fig. 3, that as the buoyancy parameter increases the concentration profiles decreases.

The heat transfer coefficient as a function of Lewis number Le for the without dispersion case $Pe_\gamma = Pe_\xi = 0$ is plotted in Fig. 4. From this figure it can be seen that an increase in the value of the mixed convection and radiation parameter R increases the heat transfer rates. The heat transfer coefficient is observed to increase with the radiation parameter and diffusivity ratio in the opposing buoyancy case, whereas it decreases in the aiding buoyancy case. For fixed values of other parameters the magnitude of $\frac{Nu}{Pe_x^{1/2}}$ for $N = 1.0$ is higher than $N = -0.5$ for all values of Le considered in the study. This clearly indicates that the buoyancy ratio has significant effect on the heat transfer coefficient than the diffusivity ratio.

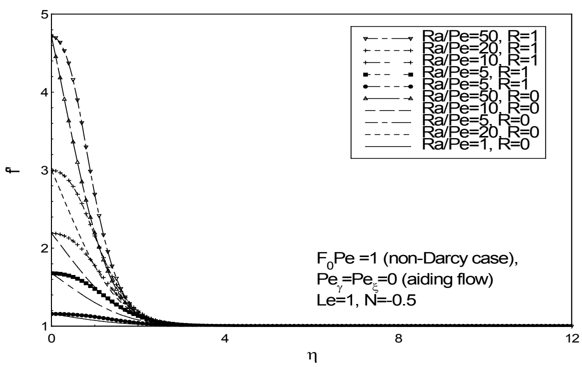

Fig.1 Non dimensional velocity profiles for varying of Ra/Pe, Le, N and R with C_T=0

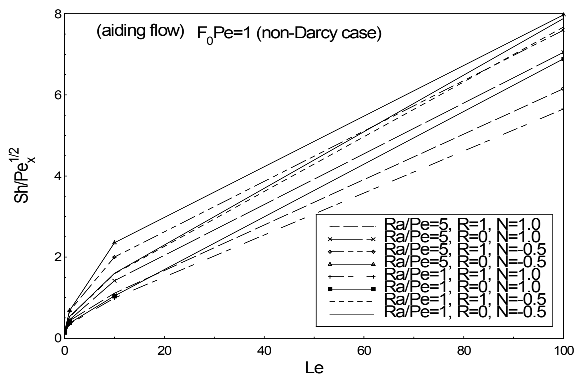

Fig.4: Heat transfer coefficient as a function of (Lewis number FPe=1, Pe_γ= Pe_ξ=0 (aiding flow

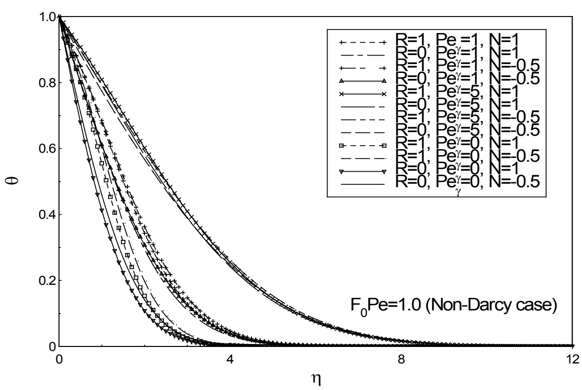

Fig.2: Variation of temperature profiles for varying N, R, and Pe_γ with Ra/Pe=1, Le=1, Pe_ξ=0

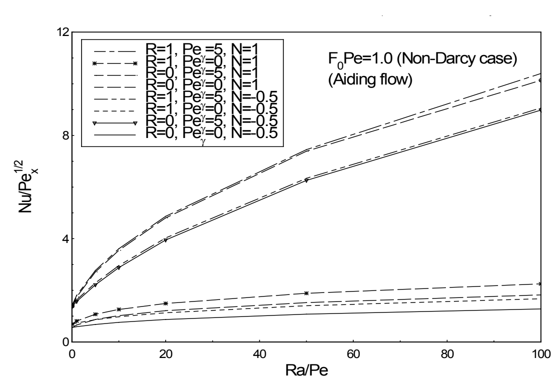

Fig.5: Mass transfer coefficient for varying Ra/Pe, R and N with Pe_γ= Pe_ξ=0, C_T=0.01

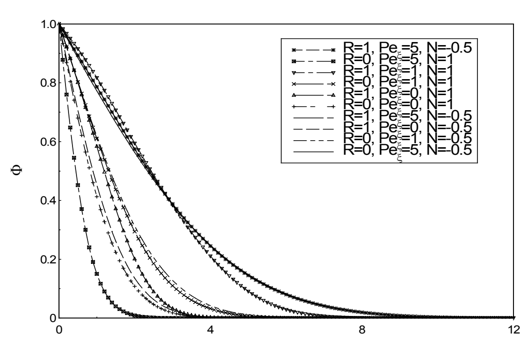

Fig. 3: Variation of concentration profiles for varying R, Pe_ξ, N with Ra/Pe=1, Pe_γ=0, Le=1, CT=0.01

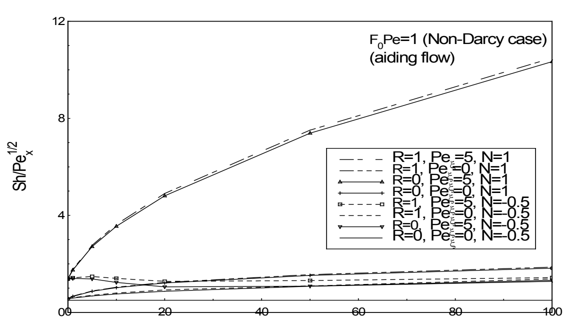

Fig.6: Variation of heat transfer coefficient for .varying R, Pe_γ, N withLe=1, Pe_ξ=0 and C_T=0.01

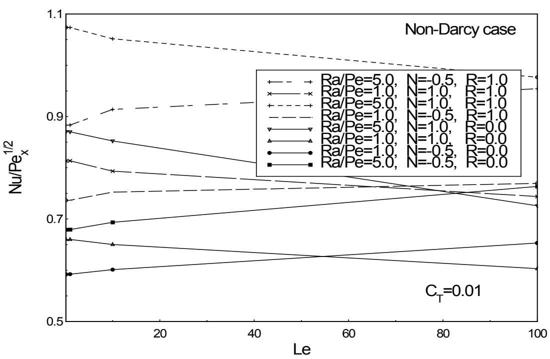

Fig.7: Variation of mass transfer coefficient for .varying R, Pe_γ=0, N withLe=1, Pe_ξ=0 and C_T=0.01

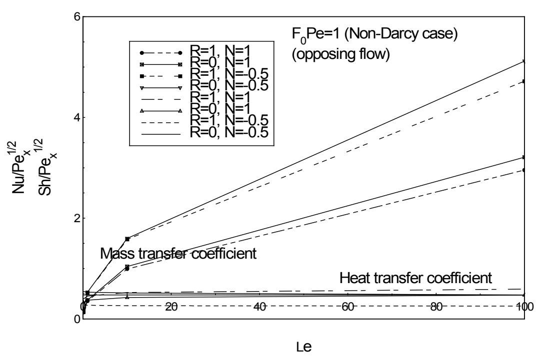

Fig.8: Variation of heat transfer coefficient for varying R, Pe_γ, N withLe=1, Pe_ξ=0 and C_T=0.01.

Fig. 5 clearly indicate the favorable effect of the Lewis number on the mass transfer coefficient in both opposing and aiding buoyancies, but the effect of radiation parameter reduced the mass transfer coefficient. Uniform trend in the Sherwood number results is observed with increase in the buoyancy ratio N from –0.5 to 1.0.

The variation of the heat transfer coefficient with Ra/Pe for nonzero values of Pe_γ is studied for a wide range of values of Le and plotted in Fig. 6. For $Le = 1.0$ this figure clearly indicate the favorable influence of thermal dispersion and radiation on the heat transfer results. The value of $-\frac{Nu}{Pe_x^{1/2}}$ increases with increasing Ra/Pe. Aiding buoyancy and radiation parameter R favors the heat transfer, whereas this favorable action is aided by $Le = 1.0$ when $N = -0.5$, and is suppressed by the $Le = 1.0$ when $N = 1.0$. These results are in agreement with the results reported by Murthy (2000) and Lai (1991).

The complex interaction between R, N, Pe_ξ and Ra/Pe show complex behavior for $-\frac{Sh}{Pe_x^{1/2}}$ curve. The result presented in Fig. 7 with $Le = 1.0$, $F_0 Pe = 1.0$ and $Pe_\gamma = 0$. In the case of aiding buoyancy, when $Le = 1.0$, the mass transfer coefficient increases with Ra/Pe and, the dispersion mechanism augments the mass transfer, when $N = -0.5$, $-\frac{Sh}{Pe_x^{1/2}}$ increases with R and Ra/Pe for $Pe_\xi = 0$. When $Pe_\xi = 5$, it increases with Ra/Pe upto the value 5 and the decreases thereafter. Its value becomes less than the corresponding value for $Pe_\xi = 1.0$ from $Ra/Pe = 20$ on wards: it may be inferred that the strength of the solutal dispersion becomes insignificant at higher values of Ra/Pe in the case of opposing buoyancy.

Opposing flow. The flow field becomes more complex when the freestream flow is opposing the buoyancy. Like in the aiding flow case, the wall velocity depends only the inertial, radiation parameters and buoyancy ratio. Flow separation is the most common ratio. Flow separation is the most common feature observed in the opposing flows. The flow separation point also depends on the buoyancy ratio. In the Forchheimer flow ($F_0 Pe = 1$) the occurrence of the flow separation is delayed the separation points are observed to occur at $Ra/Pe = 0.1$, 1 for $N = -0.5$, 1.0 and $R = 0$, 0.5, 1.0. The presence of the radiation, thermal and solutal dispersion diffusivity will not alter the point of flow separation in non-Darcy flow. The heat and mass transfer coefficients in opposing flow are presented in Fig. 8. As expected the heat transfer decreases with Le for opposing buoyancy, where as it increases with Le for aiding buoyancy. It is just a reverse mechanism to the aiding flow case and is clearly seen also, in Fig. 8. The $-\frac{Sh}{Pe_x^{1/2}}$ values for the opposing buoyancy are at higher level than those for aiding buoyancy. It is evident from the Fig. 8 that the mass transfer coefficient increases with Le also, the $-\frac{Sh}{Pe_x^{1/2}}$ values in opposing buoyancy are higher level than those in aiding buoyancy. The radiation parameter enhance the heat transfer coefficient and reduce the mass transfer coefficient. Aiding buoyancy is hindrance to the freestream flow in the opposing flow case, so a reduction in the transport quantities is observed.

Concluding Remarks

Similarity solution for hydrodynamic dispersion-radiation in mixed convection heat and mass transfer near vertical surface embedded in a porous medium has been presented. The heat and mass transfer in the boundary layer region has been analyzed for aiding and opposing buoyancies in both the aiding and opposing flows. The structure of the flow, temperature and concentration fields in the non-Darcy porous media are governed by complex interactions among the diffusion rate Le and buoyancy ratio N in addition to the flow driving parameter Ra/Pe. For small values of Le in the opposing buoyancy, flow reversal near the wall is observed. The heat transfer coefficient always increases with Ra/Pe. Thermal dispersion-radiation favors the heat transfer. As Le increases the effect of solutal dispersion on the non-dimensional mass transfer coefficient becomes less predictable in both aiding and opposing buoyancies. In the opposing flow case, the flow separation point is observed to depend on the inertial parameter and buoyancy ratio. A reduction in the heat and mass transfer coefficients is seen with increasing values of Ra/Pe. The Lewis number has complex impact on the heat and mass transfer mechanism.

Nomenclature

c inertial coefficient

C concentration

C_T temperature ratio

d particle diameter

D mass diffusivity

D_C effective mass diffusivity

f dimensionless stream function

$F_0 Pe$ parameter representing non-Darcian effects

g acceleration due to gravity

k thermal conductivity

K permeability coefficient of the porous medium

Le Lewis number

N buoyancy ratio

$\frac{Nu}{Pe_x^{1/2}}$ non-dimensional heat transfer coefficient

Pe_x local Peclet number

Pe_γ parameter representing thermal dispersion effects

Pe_ξ parameter representing solutal dispersion effects

q^r radiative flux

R radiation parameter

Ra_x modified Rayleigh number

$\frac{Sh}{Pe_x^{1/2}}$ non-dimensional mass transfer coefficient

T temperature

u, v velocity components in x and directions

x, y axial and normal coordinates

α thermal diffusivity

α_{ff} effective of thermal diffusivity

β_T coefficient of thermal expansion

β_C coefficient of solutal expansion

η dimensionless distance

θ non-dimensional temperature

ν kinematic viscosity

ϕ non-dimensional concentration

γ coefficient of dispersion thern diffusivity

ξ coefficient of dispersion solu diffusivity

ψ stream function

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