Kingdom of Saudi Arabia
Ministry of Higher Education
Majmaah University
Zulfi, College of Sciences
Mathematics Department

جامعة المجمعة
Majmaah University

المملكة العربية السعودية
وزارة التعليم العالي
جامعة المجمعة
كليـة العلــــوم بالزلفي
قسم الرياضيات

# COURSE CLASSIFICATION FORM

| Course Number/Name | | MATH 472 Differential Geometry | |
|---|---|---|---|
| **Prepared by** | | **Dr. Abd El-Nasser Ghareeb** | |
| **Program Learning Outcomes** | **Levels* (0,1,2, 3,4,5)** | **Relevant Activities** | **Assessment Methods/Metrics** |
| a1. Apply fundamentals and concepts of mathematics. | 5 | - Lectures<br>- assignments | • 3 Midterm and final exam<br>• Home work |
| a2. Apply fundamentals and concepts General sciences and Computer skills. | 3 | - assignments on logic statements | • 1 Midterm and final exam<br>• Home work |
| a3. Realize Social and ethical values | 0 | | • |
| b1. Read and construct mathematical arguments and proofs. | 4 | - Lectures<br>- assignments | Home work |
| b2. Apply critical thinking skills to solve problems that can be modeled mathematically. | 5 | - Lectures<br>- assignments<br>- Oral discussion | • 3 Midterm and final exam+ Home work |
| c1. Work independently and within a team | 3 | Divided students into groups and using oral discussion with homework | • Home work |
| c2. Bear responsibility for different situations. | 2 | | • Quizzes |
| c3. Realize codes of ethics and their importance. | 0 | | |
| d1. Communicate a depth and breadth of mathematical knowledge, both orally and in writing. | 4 | - Lectures<br>- assignments<br>- Oral discussion | • 3 Midterm + final exam<br>• Home work<br>• Quizzes |
| d2. Ability to Organize, connect and communicate mathematical and algorithmic ideas. | 4 | - Lectures<br>- assignments | • Home work<br>• Quizzes |
| d3. Critically interpret numerical and graphical data. | 3 | - assignments on information data and represented data | • Home work<br>• Quizzes |
| e1. Use computer and its applications as an office tool | 3 | - assignments on Logical expression | Home work<br>Quizzes |

*** Please mark (or type) High (5), Medium-High (4), Medium (3), Low-Medium (2), Low (1) or Not At All (0) indicating the level to which you believe, as an instructor, the students have achieved these outcomes in this course.**

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# Course Objectives and Outcomes

**Course Number: MATH 472    Course Name: Introduction to Differential Geometry**
**Prepared by: Dr. Abd El-Nasser Ghareeb**
**Table 1**: Relationship of course objectives/outcomes with PLO and ASIIN Criteria

| Course Objectives: | Course Outcomes: | ASIIN | PLO |
|---|---|---|---|
| Have the knowledge of Theory of curves in R, Regular curves, arc length and reparametrization and Natural parametrization. . | **Define** and **recognize** the Theory of curves in R, Regular curves, arc length and reparametrization and Natural parametrization. | a, b, e, m | |
| | **Improve** and **outline** the logical thinking. | b, c | |
| | **Illustrate** how to communicating with: Peers, Lecturers and Community. | l, n | |
| Have the knowledge of Serret-Frenet apparatus, Existence and uniqueness theorem for space curves and Bertrand curves. | **Define** and **recognize** the Serret-Frenet apparatus, Existence and uniqueness theorem for space curves and Bertrand curves. | a, b, c, g, m , j | |
| | **Shown** the ability of working independently and with groups. | n | |
| | **Illustrate** how take up responsibility. | l, n | |
| Studying the Involutes and evolutes, Local theory of surfaces, Simple surfaces-Coordinate transformations, Tangent vectors & tangent spaces , First and second fundamental forms. | **Define** and **recognize** the Involutes and evolutes, Local theory of surfaces, Simple surfaces-Coordinate transformations, Tangent vectors & tangent spaces, First and second fundamental forms. | a, b, f, h | |
| | ability to **write** Mathematical equations in a correct mathematical way | a, j, g | |
| Studying the normal and geodesic curvature. | **Define** and **recognize** the normal and geodesic curvature. | a, c, h | |
| | **Appraise** how to Use the computer skills and library. | d, h | |
| | **Illustrate** how to Search the internet and using software programs to deal with problems | d, h | |
| Have the knowledge of Weingarten map. | **Define** and **recognize** the Weingarten map, Pricipal Gaussian and mean curvatures. | a, e, i | |
| | **interpret** how to Know the Weingarten map, Pricipal Gaussian and mean curvatures using the internet | k, h, g | |
| Studying principal Gaussian and mean curvatures | **Define** and **recognize** the principal Gaussian and mean curvatures | a, i | |
| | **interpret** how to Know the principal Gaussian | h, k | |

| | | | |
|---|---|---|---|
| | and mean curvatures using the internet | | |
| Studying the Geodesics- Equations of Gauss and Godazzi-Mainardi. | **Define** and **recognize** Geodesics- Equations of Gauss and Godazzi-Mainardi theory | a, i | |
| | **interpret** how to Know the Geodesics-Equations of Gauss and Godazzi-Mainardi using the internet | k, h, g | |

**Table 2**: Methods of assessment of course syllabus

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| Assessment Method | Number/Type | | | | Instructor Assessed | TA/Grader Assessed | Peer/Self Assessed |
|---|---|---|---|---|---|---|---|
| Homework | 5 homework assignments | | | | x | | |
| Mid Terms/Final Exams | 2 mid-term; 1 final exam | | | | x | | |
| Quizzes | One biweekly | | | | x | | |
| Individual Projects | 1-2 wks | 3-4 wks | 1/2 sem | Full sem | | | |
| Team Projects | 1-2 wks | 3-4 wks x | 1/2 sem | Full sem x | x | | x |
| Lab Assignments | | | | | | | |
| Computer Assignments | | | | | | | |
| Computer Tools Used | | | | | | | |
| Oral Presentations | one | | | | x | | x |
| Written Reports | one | | | | x | | |
| Other | Design project (project binder) | | | | x | | |

| Outcome of ASIIN | |
|---|---|
| **a** | Graduates have sound mathematical knowledge. They have a profound overview of the contents of fundamental mathematical disciplines and are able to identify their correlations. |
| **b** | Graduates are able to recognise mathematics-related problems, assess their solvability and solve them within a specified time frame. |
| **c** | Graduates have a basic ability to work in a scientific way. They are in particular able to formulate mathematical hypotheses and have an understanding of how such hypotheses can be verified or falsified using mathematical methods. |
| **d** | Graduates can flexibly apply mathematical methods of fundamental component areas of mathematics and are able to transfer the findings obtained to other component areas or applications. |
| **e** | Graduates have abstraction ability and are able to recognise analogies and basic patterns |
| **f** | Graduates are able to think in a conceptual, analytical and logical manner. |
| **g** | Graduates have an extensive comprehension of the significance of mathematical modelling. Are able to create mathematical models for mathematical problems as well as for problems in other areas of science or everyday life, and have a selection of problem solving strategies at their disposal. |
| **h** | Graduates can use basic methods of computer-aided simulation, mathematical software and programming to solve mathematical problems |
| **i** | Graduates are in a position to solve more extensive mathematical |
| **j** | Graduates can classify, recognise, formulate and solve mathematics-related problems |
| **k** | Graduates use electronic media competently |
| **l** | Graduates can implement lifelong learning strategies. A prerequisite for this is that the students are per-severing and that they have developed persistence**.** |
| **m** | Graduates can recognise, formulate, classify and solve problems in a mathematical context |
| **n** | Graduates can communicate, possibly also in a foreign language, and contribute their work effectively in teams |

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# Instructor Course Evaluation Form

The purpose of this evaluation is to collect instructor feedback for improving this course and the Mathematics program. Information will also be used for program accreditation purposes.

## I. Program Learning Outcomes Evaluations

| Course Number/Name | MATH 472 Introduction to differential Geometry | Semester | First 1434/1435 |
|---|---|---|---|
| Instructor | Dr. Abd El-Nasser Ghareeb | | |

**The course listed above is designed for students to achieve the following outcomes at a Not At All, Low, Low- Medium, Medium, Medium-High or High level.**

**Please mark (or type) High (5), Medium-High (4), Medium (3), Low-Medium (2), Low (1) or Not At All (0) indicating the level to which you believe, as an instructor, the students have achieved these outcomes in this course.**

| Program Learning Outcomes | Relevant Activities | 5 | 4 | 3 | 2 | 1 | 0 |
|---|---|---|---|---|---|---|---|
| a1. Apply fundamentals and concepts of mathematics. | - Lectures<br>- assignments | 5 | | | | | |
| a2. Apply fundamentals and concepts General sciences and Computer skills. | - assignments on logic statements | | | 3 | | | |
| a3. Realize Social and ethical values. | | | | | | | 0 |
| b1. Read and construct mathematical arguments and proofs. | - Lectures<br>- assignments | | 4 | | | | |
| b2. Apply critical thinking skills to solve problems that can be modeled mathematically. | - Lectures<br>- assignments<br>- Oral discussion | 5 | | | | | |
| c1. Work independently and within a team | Divided students into groups and using oral discussion with homework | | | 3 | | | |
| c2. Bear responsibility for different situations. | | | | | 2 | | |
| c3. Realize codes of ethics and their importance. | | | | | | | 0 |
| d1. Communicate a depth and breadth of mathematical knowledge, both orally and in writing. | - Lectures<br>- assignments<br>- Oral discussion | | 4 | | | | |
| d2. Ability to Organize, connect and communicate mathematical and algorithmic ideas. | - Lectures<br>- assignments | | 4 | | | | |
| d3. Critically interpret numerical and graphical data. | - assignments on information data and represented data | | | 3 | | | |
| e1. Use computer and its applications as an office tool | - assignments on Logical expression | | | 3 | | | |

## II. Catalog Description , and Course Prerequisites Evaluations:

**Based on your experiences in the course, please respond by circling the most appropriate number. Circle N/A for items that are not applicable, or if you have no opinion.**

| **Catalog Description 1434-1435** | • Theory of curves in R3-Regular curves, arc length and reparametrization and Natural parametrization.<br>• Serret-Frenet apparatus, Existence and uniqueness theorem for space curves and Bertrand curves.<br>• Involutes and evolutes, Local theory of surfaces, Simple surfaces and Coordinate transformations.<br>• Tangent vectors and tangent spaces, First and second fundamental forms and Normal and geodesic curvature.<br>• Weingarten map, Pricipal Gaussian and mean curvatures and Geodesics.<br>• Equations of Gauss and Godazzi-Mainardi. | | | | | | |
|---|---|---|---|---|---|---|---|
| Course Prerequisites: | PMTH 112 + PMTH127 | **Circle One (5=Strongly Agree; 1=Strongly disagree)** | | | | | |
| 2a. Do you believe that the catalog description (above) is accurate for this course? | | **(5)** | **4** | **3** | **2** | **1** | **N/A** |
| 2b. Do you believe that the course prerequisites (above) are appropriate for this course? | | **5** | **(4)** | **3** | **2** | **1** | **N/A** |
| 2c. If not, please list any prerequisites you believe are not appropriate for this course. | | | | | | | |

## III. Textbook(s) and/or Lab Manuals (if applicable) Evaluations:

| **Textbook(s) and/or Lab Manuals (if applicable):** | • R. Millman & G.Parker, Elements of differential Geometry.<br>• Manfredo Do Carmo: Differential Geometry of Curves and Surfaces, Birkhauser, Boston, 1992.<br>• Michael Spivak: Introduction to differential Geometry, Vol. 1, 3 Edition, Addison-Wesley, 1965. | **Circle One (5=Strongly Agree; 1=Strongly Disagree)** | | | | | |
|---|---|---|---|---|---|---|---|
| 3a. In general, do you believe this to be an appropriate textbook for this course? | | **(5)** | **4** | **3** | **2** | **1** | **N/A** |
| 3b. Was the organization of the textbook appropriate for this course? | | **5** | **(4)** | **3** | **2** | **1** | **N/A** |
| 3c. Was the level of the textbook appropriate for this course? | | **5** | **(4)** | **3** | **2** | **1** | **N/A** |

## IV. Computer usage (if applicable) Evaluations:

| Computer usage (if applicable): | | Circle One (5=Strongly Agree; 1=Strongly Disagree) | | | | | |
|---|---|---|---|---|---|---|---|
| 5a. Was the use of computer well integrated with the course? | | 5 | 4 | (3) | 2 | 1 | N/A |
| 5b. Was the computer lab adequately equipped with well-maintained and updated computers? | | 5 | 4 | 3 | 2 | (1) | N/A |
| 5c. Was the computer lab equipped with sufficient number of computers? | | 5 | 5 | 5 | 2 | 1 | (N/A) |
| 5d. Were the special software packages (MATLAB, SPSS, C+, FORTRAN, etc) available and accessible? | | 5 | 4 | 3 | 2 | 1 | (N/A) |
| 5e. Was adequate technical support available when needed? | | 5 | 4 | 3 | 2 | 1 | (N/A) |

## Solution Manual

**Question 1:** **[10 marks]**

(1) Find the length of the circular helix $r(u) = \mathbf{a}\cos u\,i + a\sin u\,j + c\,u\,k$, $-\infty < u < \infty$, from $(a, 0, 0)$ to $(a, 0, 2\pi c)$.

**Solution.** Clearly the limits of $u$ are from $cu = 0$ to $cu = 2\pi c$ *i.e.* from $u = 0$ to $u = 2\pi$.

The equation of the circular helix is

$$\mathbf{r}(u) = a\cos u\,\mathbf{i} + a\sin u\,\mathbf{j} + cu\,\mathbf{k}$$

$$\therefore \quad \dot{\mathbf{r}} = \frac{d\mathbf{r}}{du} = -a\sin u\,\mathbf{i} + a\cos u\,\mathbf{j} + c\,\mathbf{k}$$

$$|\dot{\mathbf{r}}(u)| = (a^2\sin^2 u + a^2\cos^2 u + c^2)^{1/2} = (a^2 + c^2)^{1/2}$$

Therefore the length of the circular helix from $(a, 0, 0)$ to $(a, 0\ 2\pi c)$ is

$$= \int_0^{2\pi} |\dot{\mathbf{r}}(u)|\,du = \int_0^{2\pi} \sqrt{(a^2 + c^2)}\,du$$

$$= \sqrt{(a^2 + c^2)}\,[u]_0^{2\pi} = 2\pi\sqrt{(a^2 + c^2)}.$$

Again suppose $s$ denotes the arc length from the point where $u = 0$ to any point $u$, we have

$$s = \int_0^u |\dot{\mathbf{r}}(u)|\,du$$

$$= \int_0^u \sqrt{(a^2 + c^2)}\,du = (a^2 + c^2)^{1/2}\,[u]_0^u = u\,(a^2 + c^2)^{1/2}$$

$$\therefore \quad u = \frac{s}{(a^2 + c^2)^{1/2}}$$

**(2)** Show that the Serret-Frenet formulae can be written in the form $t' = w \times t$, $n' = w \times n$, $b' = w \times b$.

**Solution.** w is called Darbouxe vector of the curve.
We have from Frenet's formulae

$$t' = \kappa n = \tau t \times t + \kappa b \times t \qquad [\because\ t \times t = 0, b \times t = n]$$
$$= (\tau t + \kappa b) \times t$$

$$= (\tau t + \kappa b) \times n = w \times n,$$
$$b' = -\tau n = \tau (t \times b) + \kappa (b \times b) \qquad [\because\ b \times b = 0, -n = t \times b]$$
$$= (\tau t + \kappa b) \times b = w \times n \qquad \text{where } w = \tau t + \kappa b \text{ from (1).}$$

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**Question 2:** **[10 marks]**

(1) Prove that a curve is uniquely determined except as to position in space when its curvature and torsion are given functions of its arc length (Uniqueness Theorem for space curves).

**Proof :** If possible let there be two curves $C$ and $C_1$ having equal curvature $\kappa$ and equal torsion $\tau$ for the same values of s. Let the suffix unity by used for quantities belonging to $C_1$.

Now if $C_1$ is moved (without deformation) so that the two points on $C$ and $C_1$ corresponding to same value of $s$ coincide. We have

$$\frac{d}{ds}(\mathbf{t}_1 \cdot \mathbf{t}_1) = \mathbf{t} \cdot \kappa_1 \mathbf{n}_1 + \kappa \mathbf{n} \cdot \mathbf{t}_1$$

or

$$\frac{d}{ds}(\mathbf{t} \cdot \mathbf{t}_1) = \mathbf{t} \cdot \kappa \mathbf{n}_1 + \kappa \mathbf{n} \cdot \mathbf{t}_1 \qquad [\because \kappa_1 = \kappa \text{ given}] \qquad ...(1)$$

$$\frac{d}{ds}(\mathbf{n} \cdot \mathbf{n}_1) = \mathbf{n} \cdot (\tau \mathbf{b}_1 - \kappa \mathbf{t}_1) + (\tau \mathbf{b} - \kappa \mathbf{t}) \cdot \mathbf{n}_1 \qquad ...(2)$$

$$\frac{d}{ds}(\mathbf{b} \cdot \mathbf{b}_1) = \mathbf{b}(-\tau \mathbf{n}_1) + (-\kappa \mathbf{n}) \cdot \mathbf{b}_1 \qquad ...(3)$$

Adding equations (1), (2) and (3), we get

$$\frac{d}{ds}(\mathbf{t} \cdot \mathbf{t}_1 + \mathbf{n} \cdot \mathbf{n}_1 + \mathbf{b} \cdot \mathbf{b}_1) = 0,$$

which on integrating gives

$$\mathbf{t} \cdot \mathbf{t}_1 + \mathbf{n} \cdot \mathbf{n}_1 + \mathbf{b} \cdot \mathbf{b}_1 = \text{constant}. \qquad ...(4)$$

If $C_1$ is moved in such a manner that at $s = 0$ the two triads ($\mathbf{t}, \mathbf{n}, \mathbf{b}$ and $\mathbf{t}_1, \mathbf{n}_1, \mathbf{b}_1$) coincide. Then at that point $\mathbf{t} = \mathbf{t}_1, \mathbf{n} = \mathbf{n}_1, \mathbf{b} = \mathbf{b}_1$ and then the value of constant in eqn. (4) becomes 3.

Thus

$$\mathbf{t} \cdot \mathbf{t}_1 + \mathbf{n} \cdot \mathbf{n}_1 + \mathbf{b} \cdot \mathbf{b}_1 = 3.$$

But the sum of three cosines is equal to 3 if each angle is zero or is an integral multiple of $2\pi$.

Thus for each pair of corresponding points

$$\mathbf{t} = \mathbf{t}_1, \mathbf{n} = \mathbf{n}_1, \mathbf{b} = \mathbf{b}_1$$

Also $\mathbf{t} = \mathbf{t}_1$ gives $\mathbf{r}' = \mathbf{r}_1'$

*i.e.* $\frac{d}{ds}(\mathbf{r} - \mathbf{r}_1) = 0,$ *i.e.* $\mathbf{r} - \mathbf{r}_1 = \mathbf{a}$ (const. vector)

but as $s = 0$, $\mathbf{r} - \mathbf{r}_1 = 0$ or $\mathbf{r} = \mathbf{r}_1$ at all corresponding points and hence the two curves coincide or the two curves are congruent. This theorem is called *uniqueness theorem*.

(2) If a curve lies on a sphere, show that $\boldsymbol{\rho}$ and $\boldsymbol{\sigma}$ are related by $\frac{d}{ds}(\sigma\rho') + \frac{\rho}{\sigma} = 0$ .

**Solution.** **Necessary condition :** Let the curve lie on a sphere then to prove the given condition. Now the sphere will be osculating sphere for every point. The radius $R$ of the osculating sphere is given by

$$R^2 = \rho^2 + \sigma^2\rho'^2 \quad ...(1)$$

Differentiating w.r.t. '$s$', we get

$$0 = \rho\rho' + \sigma^2\rho'\rho'' + \sigma\sigma'\rho'^2$$

Dividing by $\rho'\sigma$, we get

$$0 = \frac{\rho}{\sigma} + \rho''\sigma + \sigma'\rho'$$

$$0 = \frac{\rho}{\sigma} + \frac{d}{ds}(\sigma\rho') \quad \text{or} \quad \frac{\rho}{\sigma} + \frac{d}{ds}\left(\frac{\rho'}{\tau}\right) = 0$$

**Question 3:** **[10 marks]**

(1) Find the plane that has three point contact at the origin with the curve $x = u^4 - 1, y = u^3 - 1, z = u^2 - 1$.

**Solution.** Let the equation of the plane at the origin be

$$lx + my + nz = 0 \quad ...(1)$$

The equations of the given curve are

$$x = u^4 - 1, y = u^3 - 1, z = u^2 - 1 \quad ...(2)$$

At the origin, $u^4 - 1 = 0, u^3 - 1 = 0, u^2 - 1 = 0$

Clearly $u = 1$ satisfies all of these three equations.

∴ At the origin, we have $u = 1$.

Now the points of intersection of the curve (2) and the surface (1) are given by the zeroes of the function

$$F(u) = l(u^4 - 1) + m(u^3 - 1) + n(u^2 - 1)$$

or $$F(u) = lu^4 + mu^3 + nu^2 - l - m - n \quad ...(3)$$

For three point contact, we should have $F'(u) = 0$, $F''(u) = 0$ where $F'(u) = dF/du$.

Now $$F'(u) = 4lu^3 + 3mu^2 + 2nu = 0 \quad ...(4)$$

and $$F''(u) = 12lu^2 + 6mu + 2n = 0 \quad ...(5)$$

At the origin *i.e.* at $u = 1$, the equation (4) and (5) becomes

$$4l + 3m + 2n = 0, \qquad 12l + 6m + 2n = 0$$

Solving, $m = -(8/3)\, l, n = 2l$

Putting values in (1), the equation of the required plane is given by

$$lx - (8/3)\, ly + 2lz = 0 \quad \text{or} \quad 3x - 8y + 6z = 0$$

(2) Calculate the curvature and the torsion of the cubic curve given by $\boldsymbol{r} = (\boldsymbol{u}, \boldsymbol{u}^2, \boldsymbol{u}^3)$.

**Solution.** Here $\mathbf{r} = (u, u^2, u^3)$

$\therefore \quad \dot{\mathbf{r}} = (1, 2u, 3u^2);\ \ddot{\mathbf{r}} = (0, 2, 6u),\ \dddot{\mathbf{r}} = (0, 0, 6)$

$$\therefore \quad \dot{\mathbf{r}} \times \ddot{\mathbf{r}} = (\mathbf{i} + 2u\,\mathbf{j} + 3u^2\,\mathbf{k}) \times (2\mathbf{j} + 6u\,\mathbf{k})$$
$$= 2\mathbf{k} - 6u\,\mathbf{j} + 12u^2\,\mathbf{i} - 6u^2\,\mathbf{i}$$
$$= 6u^2\,\mathbf{i} - 6u\,\mathbf{j} + 2\mathbf{k} = (6u^2, -6u, 2)$$
$$= 2\,(3u^2, -3u, 1)$$

$$\therefore \quad |\dot{\mathbf{r}} \times \ddot{\mathbf{r}}| = 2\,(9u^4 + 9u^2 + 1)^{1/2}.$$

Also $\quad |\dot{\mathbf{r}}, \ddot{\mathbf{r}}, \dddot{\mathbf{r}}| = \dot{\mathbf{r}} \times \ddot{\mathbf{r}} \cdot \dddot{\mathbf{r}} = 2\,(3u^2, -3u, 1) \cdot (0, 0, 6)$
$$= 2\,(0 + 0 + 6) = 12$$

$$\therefore \quad \kappa = \frac{|\dot{\mathbf{r}} \times \ddot{\mathbf{r}}|}{|\dot{\mathbf{r}}|^3} = \frac{2\,(9u^4 + 9u^2 + 1)^{1/2}}{(1 + 4u^2 + 9u^4)^{3/2}} \qquad \text{...(1)}$$

and
$$\tau = \frac{[\dot{\mathbf{r}}, \ddot{\mathbf{r}}, \dddot{\mathbf{r}}]}{|\dot{\mathbf{r}} \times \ddot{\mathbf{r}}|^2} = \frac{12}{4\,(9u^4 + 9u^2 + 1)}$$

or
$$\tau = \frac{3}{(9u^4 + 9u^2 + 1)} \qquad \text{...(2)}$$

**Question 4:** **[10 marks]**

(1) If $C$ is the original curve and $C_1$ is the locus of the center of the circle of curvature. Prove that the tangent to $C_1$ lies in the normal plane at $C$.

**Proof :** Let unity as suffix be used to distinguish quantities belonging to $C_1$.

(i) If **c** is the position vector of the centre of circle of curvature of $C$ we have

$$\mathbf{c} = \mathbf{r} + \rho\mathbf{n}.$$

Differentiating w.r.t. '$s_1$'

$$\frac{d\mathbf{c}}{ds_1} = \mathbf{t}_1 = (\mathbf{r} + \rho\mathbf{n})'\frac{ds}{ds_1} \quad \text{or} \quad \mathbf{t}_1 = (\mathbf{r}' + \rho\mathbf{n}' + \rho'\mathbf{n})\left(\frac{ds}{ds_1}\right)$$

or $$\mathbf{t}_1 = [\mathbf{t} + \rho'\mathbf{n} + \rho(\tau\mathbf{b} - \kappa\mathbf{t})]\left(\frac{ds}{ds_1}\right) \qquad [\because \mathbf{n}' = \tau\mathbf{b} - \kappa\mathbf{t}]$$

or $$\mathbf{t}_1 = (\rho'\mathbf{n} + \rho\tau\mathbf{b})\left(\frac{ds}{ds_1}\right) \qquad [\because \rho\kappa = 1] \qquad ...(1)$$

*This relation shows that the tangent to* $C_1$ *lies in the plane containing* **n** *and* **b** *i.e. normal plane to C and is inclined to* **n** *at an angle* $\alpha$ *given by*

$$\tan\alpha = \frac{\rho\tau}{\rho'} = \frac{\rho}{\sigma\rho'}$$

(ii) If $\kappa$ is constant *i.e.* $\rho$ is constant, then $\rho' = 0$

$\therefore$ from equation (1) we get

$$\mathbf{t}_1 = \rho\tau\mathbf{b}\frac{ds}{ds_1} \qquad ...(2)$$

Taking module of both sides, we get

$$1 = \rho\tau\frac{ds}{ds_1} \quad i.e. \quad \frac{ds}{ds_1} = \frac{1}{\rho\tau} \qquad ...(3)$$

from (2) and (3) $\mathbf{t}_1 = \mathbf{b}$

Differentiating w.r.t. '$s_1$'

$$\frac{d\mathbf{t}_1}{ds_1} = \mathbf{b}'\frac{ds}{ds_1} \quad \text{or} \quad \frac{d\mathbf{t}_1}{ds_1} = \kappa_1\mathbf{n}_1 = -\tau\mathbf{n}\frac{ds}{ds_1}$$

or $$\kappa_1\mathbf{n}_1 = -\tau\mathbf{n}\frac{1}{\rho\tau} \quad \text{or} \quad \kappa_1\mathbf{n}_1 = -\kappa\mathbf{n}. \qquad ...(4)$$

This clearly shows that $\mathbf{n}_1$ is parallel to **n** and choosing the direction of $\mathbf{n}_1$ opposite to that of **n** such that $\mathbf{n}_1 = -\mathbf{n}$. Therefore from (4); $\kappa_1 = \kappa$.

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(2) For a spherical curve, prove that $\boldsymbol{\rho + \frac{d^2\rho}{d\psi^2} = 0}$.

**Solution.** A spherical curve means a curve lying on a sphere. We have proved in Ex. 1 (a) above that for a spherical curve,

$$\frac{d}{ds}(\sigma\rho') + \frac{\rho}{\sigma} = 0$$

or $$\frac{d}{ds}\left(\frac{ds}{d\psi}\frac{d\rho}{ds}\right) + \frac{\rho}{\sigma} = 0 \qquad \left[\because \sigma = \frac{1}{\tau} = \frac{ds}{d\psi}\right]$$

or $$\frac{d}{d\psi}\left(\frac{d\rho}{d\psi}\right)\frac{d\psi}{ds} + \rho \cdot \frac{d\psi}{ds} = 0$$

or $$\frac{d^2\rho}{d\psi^2} + \rho = 0. \qquad \left[\text{on dividing by } \frac{d\psi}{ds}\right]$$

**Proved**

<u>**أنتهت نموذج الإجابة**</u>

**جامعة المجمعة**

**كلية العلوم بالزلفي**

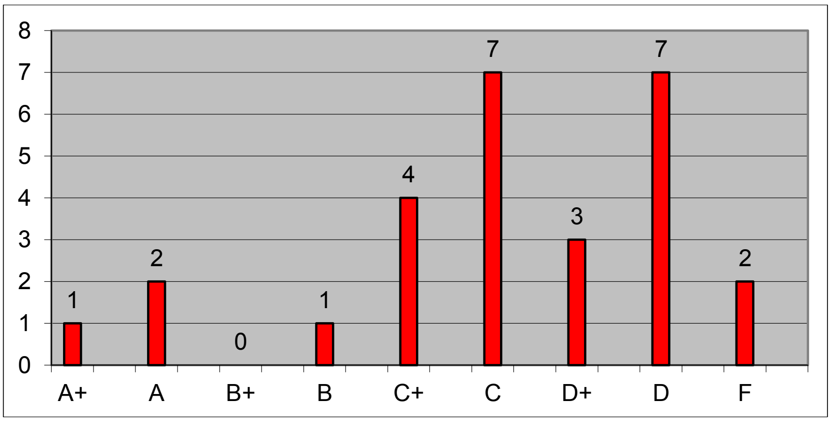
**نموذج تحويل العلامات النهائي من مئوي الى أحرف لطلبة البكالوريوس**

الفصل الدراسي **الثاني** ١٤٣٤/١٤٣٥

| | | | |
|---|---|---|---|
| **القسم** | قسم الرياضيات | **رقم المادة** | MATH 472 |
| **استاذ المادة** | د. عبدالناصر غريب عبدالرحمن | **اسم المادة** | مقدمة فى الهندسة التفاضلية |
| **عدد الطلبة المسجلين** | 27 | **عدد الطلبة الغائبين عن النهائي** | 0 |
| **عدد الطلبة الناجحين** | 25 | **عدد الطلبة الراسبين** | 2 |
| **متوسط الدرجات** | 2.85 | **العلامة الدنيا** | F |
| **الدرجة العليا** | A+ | **نسبة النجاح** | 92.59% |

| Average | Percentage | SUM | Count | TO | From | Average |
|---|---|---|---|---|---|---|
| | 3.7037037 | 5 | 1 | 100 | 95 | A+ |
| | 7.40740741 | 9.5 | 2 | 94 | 90 | A |
| | 0 | 0 | 0 | 89 | 85 | B+ |
| | 3.7037037 | 4 | 1 | 84 | 80 | B |
| | 14.8148148 | 14 | 4 | 79 | 75 | C+ |
| | 25.9259259 | 21 | 7 | 74 | 70 | C |
| | 11.1111111 | 7.5 | 3 | 69 | 65 | D+ |
| | 25.9259259 | 14 | 7 | 64 | 60 | D |
| | 7.40740741 | 2 | 2 | 59 | 0 | F |
| 2.85 | 100 | 77 | 27 | | Total Students | |

| الرقم | العلامة | التقدير |
|---|---|---|
| 1 | 61 | D |
| 2 | 63 | D |
| 3 | 47 | F |
| 4 | 60 | D |
| 5 | 72 | C |
| 6 | 68 | D+ |
| 7 | 40 | F |
| 8 | 66 | D+ |
| 9 | 70 | C |
| 10 | 62 | D |
| 11 | 72 | C |
| 12 | 91 | A |
| 13 | 75 | C+ |
| 14 | 60 | D |
| 15 | 72 | C |
| 16 | 81 | B |
| 17 | 65 | D+ |
| 18 | 96 | A+ |
| 19 | 73 | C |
| 20 | 90 | A |
| 21 | 75 | C+ |
| 22 | 77 | C+ |
| 23 | 71 | C |
| 24 | 77 | C+ |
| 25 | 72 | C |
| 26 | 60 | D |
| 27 | 60 | D |



Kingdom of Saudi Arabia
Ministry of Higher Education
Majmaah University
Zulfi, College of Sciences
Mathematics Department

المملكة العربية السعودية
وزارة التعليم العالي
جامعة المجمعة
كليـــة العلـــــــوم بالزلفي
قسم الرياضيات

# Student Course Evaluation Form

The purpose of this evaluation is to collect instructor feedback for improving this course and the Mathematics program. Information will also be used for program accreditation purposes.

## I. Program Learning Outcomes Evaluations

| | | | |
|---|---|---|---|
| Course Number/Name | MATH 472 Introduction to Differential Geometry | Semester | Second 1434/1435 |
| Instructor | **Dr. Abd El-Nasser Ghareeb** | | |
| **Student Name** | ------------------------------------ | **Student ID** | ---------------------- |

**The course listed above is designed for students to achieve the following outcomes at a Not At All, Low, Low- Medium, Medium, Medium-High or High level.**

**Please mark (or type) High (5), Medium-High (4), Medium (3), Low-Medium (2), Low (1) or Not At All (0) indicating the level to which you believe, as an instructor, the students have achieved these outcomes in this course.**

| Program Learning Outcomes | 5 | 4 | 3 | 2 | 1 | 0 |
|---|---|---|---|---|---|---|
| a1. Apply fundamentals and concepts of mathematics. | | | | | | |
| a2. Apply fundamentals and concepts General sciences and Computer skills. | | | | | | |
| a3. Realize Social and ethical values. | | | | | | |
| b1. Read and construct mathematical arguments and proofs. | | | | | | |
| b2. Apply critical thinking skills to solve problems that can be modeled mathematically. | | | | | | |
| c1. Work independently and within a team | | | | | | |
| c2. Bear responsibility for different situations. | | | | | | |
| c3. Realize codes of ethics and their importance. | | | | | | |
| d1. Communicate a depth and breadth of mathematical knowledge, both orally and in writing. | | | | | | |
| d2. Ability to Organize, connect and communicate mathematical and algorithmic ideas. | | | | | | |
| d3. Critically interpret numerical and graphical data. | | | | | | |
| e1. Use computer and its applications as an office tool | | | | | | |

## II. Catalog Description , and Course Prerequisites Evaluations:

**Based on your experiences in the course, please respond by circling the most appropriate number. Circle N/A for items that are not applicable, or if you have no opinion.**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Catalog Description 1434-1435** | • Theory of curves in R3-Regular curves, arc length and reparametrization and Natural parametrization.<br>• Serret-Frenet apparatus, Existence and uniqueness theorem for space curves and Bertrand curves.<br>• Involutes and evolutes, Local theory of surfaces, Simple surfaces and Coordinate transformations.<br>• Tangent vectors and tangent spaces, First and second fundamental forms and Normal and geodesic curvature.<br>• Weingarten map, Pricipal Gaussian and mean curvatures and Geodesics.<br>• Equations of Gauss and Godazzi-Mainardi. | | | | | | |
| Course Prerequisites: | PMTH 112 + PMTH127 | **Circle One (5=Strongly Agree; 1=Strongly disagree)** | | | | | |
| 2a. Do you believe that the catalog description (above) is accurate for this course? | | **5** | **4** | **3** | **2** | **1** | **N/A** |
| 2b. Do you believe that the course prerequisites (above) are appropriate for this course? | | **5** | **4** | **3** | **2** | **1** | **N/A** |
| 2c. If not, please list any prerequisites you believe are not appropriate for this course. | | | | | | | |

## III. Textbook(s) and/or Lab Manuals (if applicable) Evaluations:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Textbook(s) and/or Lab Manuals (if applicable):** | • R. Millman & G.Parker, Elements of differential Geometry.<br>• Manfredo Do Carmo: Differential Geometry of Curves and Surfaces, Birkhauser, Boston, 1992.<br>• Michael Spivak: Introduction to differential Geometry, Vol. 1, 3 Edition, Addison-Wesley, 1965. | **Circle One (5=Strongly Agree; 1=Strongly Disagree)** | | | | | |
| 3a. In general, do you believe this to be an appropriate textbook for this course? | | **5** | **4** | **3** | **2** | **1** | **N/A** |
| 3b. Was the organization of the textbook appropriate for this course? | | **5** | **4** | **3** | **2** | **1** | **N/A** |
| 3c. Was the level of the textbook appropriate for this course? | | **5** | **4** | **3** | **2** | **1** | **N/A** |

## IV. Computer usage (if applicable) Evaluations:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Computer usage (if applicable):** | | **Circle One (5=Strongly Agree; 1=Strongly Disagree)** | | | | | |
| 4a. Was the use of computer well integrated with the course? | | **5** | **4** | **3** | **2** | **1** | **N/A** |
| 4b. Was the computer lab adequately equipped with well- | | **5** | **4** | **3** | **2** | **1** | **N/A** |

| | | | | | | |
|---|---|---|---|---|---|---|
| maintained and updated computers? | | | | | | |
| 4c. Was the computer lab equipped with sufficient number of computers? | **5** | **5** | **5** | **2** | **1** | **N/A** |
| 4d. Were the special software packages (MATLAB, SPSS, C+, FORTRAN, etc) available and accessible? | **5** | **4** | **3** | **2** | **1** | **N/A** |
| 4e. Was adequate technical support available when needed? | **5** | **4** | **3** | **2** | **1** | **N/A** |