Kingdom of Saudi Arabia
Ministry of Higher Education
Majmaah University Faculty of Science-Zulfi
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جامعة المجمعة
Majmaah University

Advanced Mathematics For CS

"Discrete Mathematics"

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Lecture Two

1.12 Conditional Statement

A Real Life Implication

The conditional truth table is a little harder to define than the tables in the previous section. To see how to define the conditional truth table, let us analyze a statement made by a politician, Senator Bridget Terry:

If I am elected, then taxes will go down.

As before, there are four possible combinations of truth values for the two component statements. Let p represent "I am elected," and let q represent "Taxes will go down."

As we analyze the four possibilities, it is helpful to think in terms of the following: "Did Senator Terry lie?" If she lied, then the conditional statement is considered false; if she did not lie, then the conditional statement is considered true.

Possibility	Elected?	Taxes Go Down?	
1	Yes	Yes	p is T, q is T
2	Yes	No	p is T, q is F
3	No	Yes	p is F, q is T
4	No	No	p is F, q is F

1.12 Conditional Statement

A Real Life Implication

The four possibilities are as follows:

1. In the first case assume that the senator was elected and taxes did go down (p is T, q is T). The senator told the truth, so place T in the first row of the truth table. (We do not claim that taxes went down *because* she was elected; it is possible that she had nothing to do with it at all.)
2. In the second case assume that the senator was elected and taxes did not go down (p is T, q is F). Then the senator did not tell the truth (that is, she lied). So we put F in the second row of the truth table.
3. In the third case assume that the senator was defeated, but taxes went down anyway (p is F, q is T). Senator Terry did not lie; she only promised a tax reduction if she were elected. She said nothing about what would happen if she were not elected. In fact, her campaign promise gives no information about what would happen if she lost. Since we cannot say that the senator lied, place T in the third row of the truth table.
4. In the last case assume that the senator was defeated but taxes did not go down (p is F, q is F). We cannot blame her, since she only promised to reduce taxes if elected. Thus, T goes in the last row of the truth table.

1.12.A Conditional Statement: $p \rightarrow q$

تورط – تضمن - استلزام

"If he is the governor, then many schools will be built" : If p Then q / $p \rightarrow q$
The statement is true in all cases except when he is the governor and no schools were built

The properties:

1. Idempotence.
2. Commutativity.
3. Associativity.

Do not hold for Implication.

TABLE 5 **The Truth Table for the Conditional Statement $p \rightarrow q$.**

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Conditional Statement:

- p is called the premise/evidence/Basis/Foundation/Idea/hypothesis/antecedent),
- q is called the conclusion/consequent)
- $p \rightarrow q$ is false when p is true and q is false. True otherwise

1.12.A Conditional Statement $p \to q$

Variety of ways to express the connectives that appear in compound statement

Symbolic Statement	English Statement	P: A person is a father. q: A person is a male.
$p \to q$	IF p THEN q.	If a person is a father, then that person is a male.
$p \to q$	q IF p.	A person is a male, if that person is a father.
$p \to q$	p Is Sufficient For q.	Being a father is sufficient for being a male.
$p \to q$	q Is Necessary For p.	Being a male is necessary for being a father.
$p \to q$	p Only If q.	A person is a father only if that person is a male .
$p \to q$	Only If q, p.	Only if a person is a male, is that person is a father .

1.12.A Conditional Statement $p \rightarrow q$

- Variety of ways to express an implication

- if p, then q
- if p, q
- p is sufficient for q
- From p follows q
- a sufficient condition for q is p
- p only if q

p implies q

q if p

q is necessary for p

q whenever p, q when p

q follows from p

a necessary condition for p is q

q unless $\neg$ p

TABLE 5 The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Conditional Statement: $p \rightarrow q$ is false when p is true and q is false. True otherwise

<u>Example</u>

p: you go, q: I go. P → q means "If you go, then I go" is equivalent to p only if q ("You go only if I go")

(not the same as "I go only if you go" which is q only if p)

1.12.A Conditional Statement

$p \rightarrow q$

- p only if q:
  - The statement is true, hence p cannot be true when q is not true
  - The statement is false if p is true but q is false
  - When p is false, q may be either true or false
  - Not to use "q only if p" to express $p \rightarrow q$

TABLE 5 **The Truth Table for the Conditional Statement $p \rightarrow q$.**

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

- q unless $\neg$ p : مالم
  - If $\neg$ p is false, then q must be true
  - The statement is false when p is true but q is false, but the statement is true otherwise.

1.12.A Conditional Statement $p \rightarrow q$

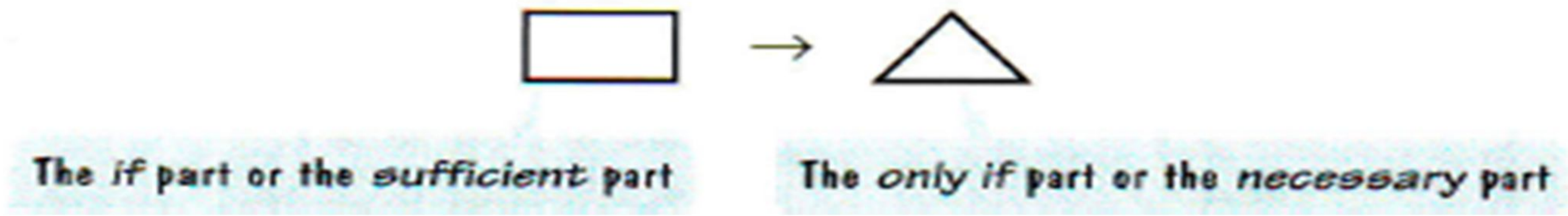
An Illustrative Example: Variety of ways to express the connectives that appear in compound statement.

The statement: If you are 18, then you can vote.

can be written in any of the following ways.

1. You can vote if you are 18.
2. You are 18 only if you can vote.
3. Being able to vote is necessary for you to be 18.
4. Being 18 is sufficient for being able to vote.
5. All 18-year-olds can vote.
6. Being 18 implies that you can vote.

1.12.A Conditional Statement p → q Necessary And Sufficient Conditions



- A **Necessary Condition** شرط الضرورة is just that:

A condition that is necessary for a particular outcome to be achieved. The condition does not grantee the outcome; but, if the condition does not hold, the outcome will not be achieved.

- A **Sufficient Condition** is just that:

A condition that suffices to grantee a particular outcome to be achieved. The condition does not grantee the outcome. If the condition does not hold, the outcome might be achieved in other ways or it might not be achieved at all; but if the condition does hold, the outcome is guaranteed.

1.12.A Conditional Statement

$p \rightarrow q$

- **If Ali learns discrete mathematics, then he will find a good job.**
  - Ali will find a good job is necessary to occur when he learns discrete mathematics (q when p).
  - For Ali to get a good job, it is sufficient for him to learn discrete mathematics (sufficient condition for q is p).
  - Ali will find a good job unless he does not learn ما لم يكن قد أقدم على عدم تعلم discrete mathematics (q unless not p).

1.12.A Conditional Statement $p \to q$

Variety of ways to express the connectives that appear in compound statement

Conditional statements in English often omit the word then and simply use a comma. When then is included, the comma can be included or omitted. Here are some examples:

- If a person is a father, then that person is a male.
- If a person is a father then that person is a male.
- If a person is a father, that person is a male.

Exercise Set 1.12.A

1. Many students have difficulty interpreting *necessary* and *sufficient*. Use the statement "Being in Canada is sufficient for being in North America" to explain why "p is sufficient for q" translates as "if p, then q."

2. Use the statement "To be an integer, it is necessary that a number be rational" to explain why "p is necessary for q" translates as "if q, then p."

Exercise Set 1.12.A

3. Exercises 1-8, let p and q represent the following simple statements:

p: This is an alligator تمساح أمريكي. & q: This is a reptile زاحف.

Write each of the following compound statements in symbolic form.

1. If this is an alligator, then this is a reptile.
2. If this is a reptile, then this is an alligator.
3. If this is not an alligator, then this is not a reptile.
4. If this is not a reptile, then this is not an alligator.
5. This is not an alligator if it's not a reptile.
6. This is a reptile if it's an alligator.
7. Being a reptile is necessary for being an alligator.
8. Being an alligator is sufficient for being a reptile.

Exercise Set 1.12.A

4. Exercises 1-8, let p and q represent the following simple statements:

p: You are human. & q: You have feathers ريش.

Write each of the following compound statements in symbolic form.

1. You do not have feathers if you are human.
2. You are not human if you have feathers.
3. Not being human is necessary for having feathers.
4. Not having feathers is necessary for being human.
5. Being human is sufficient for not having feathers.
6. Having feathers is sufficient for not being human.
7. You have feathers only if you're not human.
8. You're human only if you do not have feathers.

1.12.B Conditional Statement p → q Hidden In An Everyday Expression (Distract)

It must be emphasized that the use of the conditional connective in no way implies a cause-and-effect relationship. Any two statements may have an arrow placed between them to create a compound statement. For example,

Example If I pass mathematics, then the sun will rise the next day
is true, since the consequent is true.

The conditional connective may not always be explicitly stated. That is, it may be "hidden" in an everyday expression. For example, the statement

Big girls don't cry

can be written in *if* ... *then* form as

If you're a big girl, then you don't cry.

As another example, the statement

It is difficult to study when you are distracted

can be written

If you are distracted, then it is difficult to study.

Exercise Set 1.12.B

1. Rewrite each statement using the If … Then connective. Rearrange the wording or add words as necessary

1. It must be alive if it is breathing.
2. You can believe it if you see it on the Internet.
3. Lorri Morgan visits Hawaii every summer.
4. Tom Shaffer's area code is 216.
5. Every picture tells a story.
6. All marines love boot camp.
7. No guinea pigs are scholars.
8. No koalas live in Texas.
9. Running Bear loves Little White Dove.
10. An opium-eater cannot have self-command.
11. All nurses wear white shoes.

Exercise Set 1.12.B

12. If it is muddy, I'll wear my galoshes.
13. If I finish studying, I'll go to the party.
14. "17 is positive" implies that 17 + 1 is positive.
15. "Today is Wednesday" implies that yesterday was Tuesday.
16. All integers are rational numbers.
17. All whole numbers are integers.

Exercise Set 1.12.B

18. Doing crossword puzzles is sufficient for driving me crazy.
19. Being in Fort Lauderdale is sufficient for being in Florida.
20. A day's growth of beard is necessary for Greg Tobin to shave.
21. Being an environmentalist is necessary for being elected.
22. I can go from Boardwalk to Connecticut Avenue only if I pass GO.
23. The principal will hire more teachers only if the school board approves.

Exercise Set 1.12.B

24. No whole numbers are not integers.
25. No integers are irrational numbers.
26. The Indians will win the pennant when their pitching improves.
27. Jesse will be a liberal when pigs fly.
28. A rectangle is a parallelogram with a right angle.
29. A parallelogram is a four-sided figure with opposite sides parallel.
30. A triangle with two sides of the same length is isosceles.
31. A square is a rectangle with two adjacent sides equal.
32. The square of a two-digit number whose units digit is 5 will end in 25.
33. An integer whose units digit is 0 or 5 is divisible by 5.

1.12.C Conditional Statement
$p \rightarrow q$

Special Characteristics of Conditional Statements

1. $p \rightarrow q$ is false only when the antecedent is *true* and the consequent is *false*.
2. If the antecedent is *false*, then $p \rightarrow q$ is automatically *true*.
3. If the consequent is *true*, then $p \rightarrow q$ is automatically *true*.

Writing a Conditional as an "or" Statement

$p \rightarrow q$ is equivalent to $\sim p \vee q$.

Negation of $p \rightarrow q$

The negation of $p \rightarrow q$ is $p \wedge \sim q$.

Exercise Set 1.12.C

1. Write the negation of each of the following Statements.

1. If that is an authentic Persian rug, I'll be surprised.
2. If Ella reaches that note, she will shatter glass.
3. If the English measures are not converted to metric measures, then the spacecraft will crash on the surface of Mars.
4. If you say "I do," then you'll be happy for the rest of your life.
5. "If you want to be happy for the rest of your life, never make a pretty woman your wife." *Jimmy Soul*
6. If loving you is wrong, I don't want to be right.

Exercise Set 1.12.C

2. Write the negation of each of the following Statements.

1. If I am in Los Angeles, then I am in California.
2. If I am in Houston, then I am in Texas.
3. If it is purple, then it is not a carrot.
4. If the TV is playing, then I cannot concentrate.
5. If he doesn't, I will.
6. If she says yes, he says no.
7. If there is a blizzard, then all schools are closed.
8. If there is a tax cut, then all people have extra spending money.
9. $\neg q \rightarrow \neg r$
10. $\neg p \rightarrow r$

1.12.D Conditional Statement $p \rightarrow q$
Important Notes

- **N.B.1**

  A conditional proposition that is true because the hypothesis is false is said to be true by default or vacuously ≡ stupidly ≡ dimly ≡ darkly true .

- **N.B.2**

  The "if" part of the sentence and the "then" part of the sentence need not be related in any intuitive sense. The truth or falsity of an "if-then" statement is simply a fact about the logical values of its hypothesis and of its conclusion. (Wit ≡ Smartness ≡ Intelligence)

- **N.B.3**

  The proposition $(\sim A) \vee B$ says the same thing as $A \rightarrow B$.

- **N.B.4**

  Common Mistake for $p \rightarrow q$:
  - Correct: p only if q .
  - Mistake to think "q only if p" .

Exercise Set 1.12.D

1. Decide whether each of the following statement is true or false.

1. If the antecedent of a conditional statement is false, the conditional statement is true.
2. If the consequent of a conditional statement is true, the conditional statement is true.
3. If q is true, then $(p \wedge q) \rightarrow q$ is true.
4. If p is true, then $\sim p \rightarrow (q \vee r)$ is true.
5. The negation of "If pigs fly, I'll believe it" is "If pigs don't fly, I won't believe it."
6. The statements "If it flies, then it's a bird" and "It does not fly or it's a bird" are logically equivalent.
7. Given that $\sim p$ is true and q is false, the conditional $p \rightarrow q$ is true.
8. Given that $\sim p$ is false and q is false, the conditional $p \rightarrow q$ is true.
9. In a few sentences, explain how to determine the truth value of a conditional statement.
10. Explain why the statement "If $3 = 5$, then $4 = 6$" is true.

Exercise Set 1.12.D

2. Let T represents a true statement and F represents a false statement, tell whether each of the following conditional is true or false.

1. $F \to (4 \neq 7)$
2. $F \to (3 \neq 3)$
3. $T \to (6 < 3)$
4. $(4 = 11 - 7) \to (8 > 0)$
5. $(6 \geq 6) \to F$
6. $(4^2 \neq 16) \to (4 - 4 = 8)$

3. Let s represent "She has a snake for a pet", p represent "He trains ponies," and m represent "They raise monkeys." Express each of the following compound statement in words.

1. $\sim m \to p$
2. $(s \wedge p) \to m$
3. $p \to \sim m$
4. $\sim p \to (\sim m \vee s)$
5. $s \to (m \wedge p)$
6. $(\sim s \vee \sim m) \to \sim p$

Exercise Set 1.12.D

4. Let b represent "I ride my bike," r represent "it rains," and p represent "the play is cancelled." Write each of the following compound statement in symbols.

1. If it rains, then I ride my bike.
2. If I ride my bike, then the play is cancelled.
3. If I do not ride my bike, then it does not rain.
4. If the play is cancelled, then it does not rain.
5. I ride my bike, or if the play is cancelled then it rains.
6. The play is cancelled, and if it rains then I do not ride my bike.
7. I'll ride my bike if it doesn't rain.
8. It rains if the play is cancelled.

5. Let p and r are False, and q is true. Find the truth value of each of the following statements.

1. $\sim r \to q$
2. $\sim p \to \sim r$
3. $q \to p$
4. $\sim r \to p$
5. $p \to q$
6. $\sim q \to r$
7. $\sim p \to (q \wedge r)$
8. $(\sim r \vee p) \to p$
9. $\sim q \to (p \wedge r)$
10. $(\sim p \wedge \sim q) \to (p \wedge \sim r)$
11. $(p \to \sim q) \to (\sim p \wedge \sim r)$
12. $(p \to \sim q) \wedge (p \to r)$

Exercise Set 1.12.D

6. Explain Why, if we know that p is true, we also know that $[r \vee (p \vee s)] \rightarrow (p \vee q)$ is true, even if we are not given the truth values of q, r, and s.

7. Construct a true statement involving a conditional, a conjunction, a disjunction, and a negation (not necessarily in that order), that consists of component statements p, q, and r, with all of these component statements False .

Exercise Set 1.12.D

8. Use truth tables to decide which of the pairs of statements are equivalent.

1. $p \to q; \quad \sim p \vee q$
2. $\sim(p \to q); \quad p \wedge \sim q$
3. $p \to q; \quad \sim q \to \sim p$
4. $q \to p; \quad \sim p \to \sim q$
5. $p \to \sim q; \quad \sim p \vee \sim q$
6. $p \to q; \quad q \to p$
7. $p \wedge \sim q; \quad \sim q \to \sim p$
8. $\sim p \wedge q; \quad \sim p \to q$

9. Write each statement as an equivalent statement that does not use the IF … Then connective. Remember that $p \to q$ is equivalent to $\neg p \vee q$.

1. If you give your plants tender, loving care, they flourish.
2. If the check is in the mail, I'll be surprised.
3. If she doesn't, he will.
4. If I say yes, she says no.
5. All residents of Butte are residents of Montana.
6. All women were once girls.

1.12.E Inverse, Converse and Contrapositive

- For conditional statement $p \rightarrow q$
  - Inverse المعكوس : $\neg p \rightarrow \neg q$
  - Converse المضاد : $q \rightarrow p$
  - Contrapositive المعكوس الموضعي : $\neg q \rightarrow \neg p$

N.B.

- Contrapositive and Conditional Statement are equivalent.
- Converse and Inverse are equivalent.

1.12.E Inverse, Converse and Contrapositive

Example 1 : Given the direct statement:

- If I live in Beraydah, then I live In Kasseem.
- Let p represent: I live in Beraydah.
- Let q represent: I live in Kasseem.

Hence:

i. The direct statement is:

$$p \to q$$

ii. The Inverse is:

$$\neg p \quad \to \quad \neg q$$

i.e. If I don't live in Beraydah, then I don't live in Kasseem.
Which is not necessarily true.

1.12.E Inverse, Converse and Contrapositive

iii. The Converse is:

$$q \rightarrow p$$

i.e. If I live in Kasseem, then I live in Beraydah.

Which is not necessarily true, even though the direct statement is.

iv. The Contrapositive is:

$$\neg q \rightarrow \neg p$$

i.e. If I don't live in Kasseem, then I don't live in Beraydah.

Which is true, like the direct statement.

- N.B.

It is clear that the Inverse and Converse of a true statement need not be true. They ***can*** be, but they need not be

1.12.E Inverse, Converse and Contrapositive

Example 2 : Given the direct statement:

- If it is a BMW, it is a car.
- Let p represent: it is a BMW & Let q represent: it is a car.

Hence:

i. The direct statement is:

$p \to q$ If it is a BMW, it is a car. (Consider it to be True)

ii. The Inverse is:

$\neg p \to \neg q$ If it is not a BMW, it isn't a car. (Not necessarily true)

iii. The Converse is:

$q \to p$ If it is a car, it is a BMW. (Not necessarily true)

iv. The Contrapositive is:

$\neg q \to \neg p$ If it isn't a car, it isn't a BMW. (Necessarily true)

1.12.E Inverse, Converse and Contrapositive

Equivalent

Equivalent

p	q	Conditional $p \to q$	Converse $q \to p$	Inverse $\sim p \to \sim q$	Contrapositive $\sim q \to \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

Not equivalent

Not equivalent

Exercise Set 1.12.E

I. For each given direct statement, write (a) the *converse*, (b) the inverse, and (c) the *contrapositive* in If … Then form. In some of the exercises, it may be helpful to restate the direct statement in *if … then,* form

1. If beauty were a minute, then you would be an hour.
2. If you lead, then I will follow.
3. If it ain't broke, don't fix it.
4. If I had a nickel for each time that happened, I would be rich.
5. Walking in front of a moving car is dangerous to your health.
6. Milk contains calcium.
7. Birds of a feather flock together.
8. A rolling stone gathers no moss.
9. If you build it, he will come.
10. Where there's smoke, there's fire.

Exercise Set 1.12.E Cont.

11. $p \rightarrow \sim q$

12. $\sim p \rightarrow q$

13. $\sim p \rightarrow \sim q$

14. $\sim q \rightarrow \sim p$

15. $p \rightarrow (q \vee r)$ (*Hint:* Use one of De Morgan's laws as necessary.)

16. $(r \vee \sim q) \rightarrow p$ (*Hint:* Use one of De Morgan's laws as necessary.)

17. State the contrapositive of "If the square of a natural number is even, then the natural number is even." The two statements must have the same truth value. Use several examples and inductive reasoning to decide whether both are true or both are false.

Exercise Set 1.12.E

II. In Exercises 1 - 12, write (a) the *converse*, (b) the inverse, and (c) the *contrapositive* of each statement.

1. If I am in Chicago, then I am in Illinois.
2. If I am in Birmingham, then I am in the South.
3. If the stereo is playing, then I cannot hear you.
4. It is blue, then it is not an apple.
5. "If you don't laugh, you die." (humorist Alan King)
6. "If it doesn't fit, you must acquit." (lawyer Johnnie Cochran)
7. If the president is telling the truth, then all troops were withdrawn.
8. If the review session is successful then no students fail the test.
9. If all institutions place profit above human need, then some people suffer.
10. If all hard workers are successful then some people are not hard workers.
11. $\neg q \to \neg r$
12. $\neg p \to r$

Exercise Set 1.12.E

In Exercises 1 - 4, express each statement in "if ... then" form. (More than one correct wording in "if . . . then" form is possible.) Then write the statement's converse, inverse, contrapositive, and negative.

1. No pain is sufficient for no gain.

2. Not observing the speed limit is necessary for getting a speeding ticket.

3. Being neither hedonistic nor ascetic are necessary for following Buddha's "Middle Way."

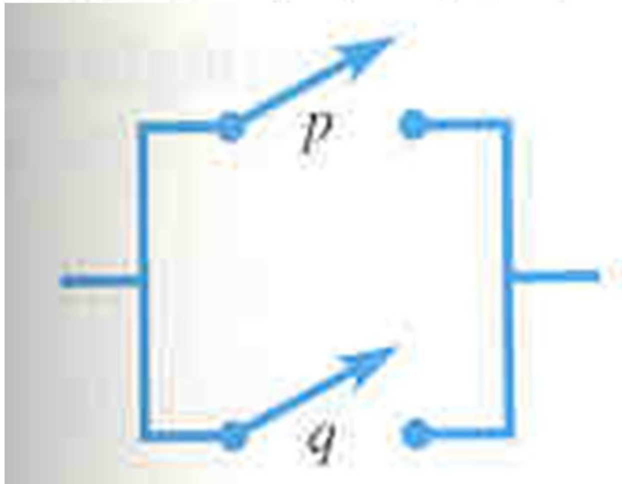
4. Going into heat and not finding a mate are sufficient for a female ferrat's death .

1.13 Transformation of Logic Expression To Switching Circuits And Vice Versa

Circuits One of the first nonmathematical applications of symbolic logic was seen in the master's thesis of Claude Shannon in 1937. Shannon showed how logic could be used to design electrical circuits. His work was immediately used by computer designers. Then in the developmental stage, computers could be simplified and built for less money using the ideas of Shannon.

Series circuit

conjunction $p \wedge q$.

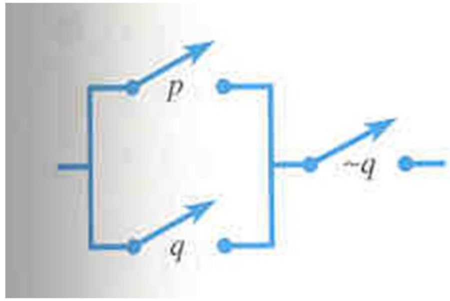


Parallel circuit

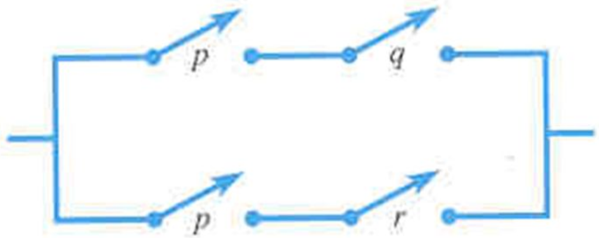
A circuit corresponding to the disjunction $p \vee q$

1.13 Transformation of Logic Expression To Switching Circuits And Vice Versa (Examples)

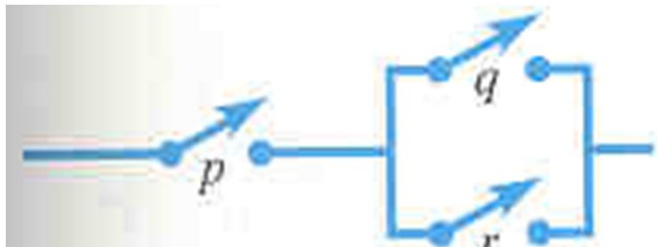
$(p \vee q) \wedge \sim q.$



$(p \wedge q) \vee (p \wedge r).$



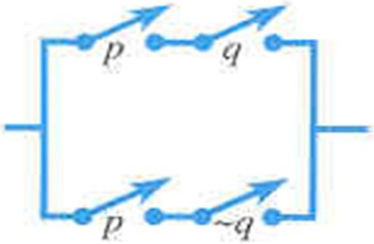
([redacted] $\vee\ r)$



Exercise Set 1.13

1. Write a logical statement representing each of the following circuits. Simplify each circuit when possible.

1.



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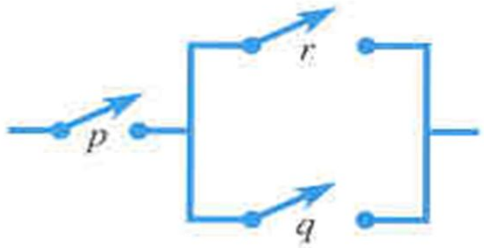

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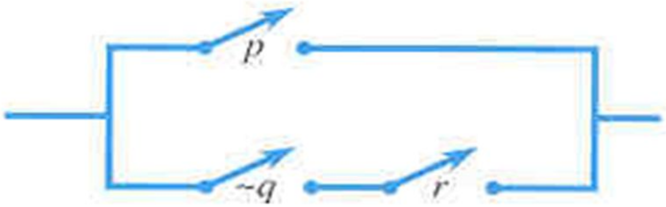
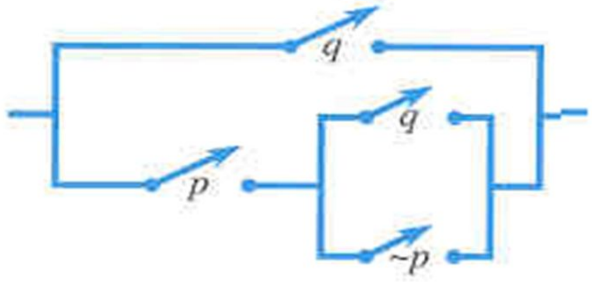


4.



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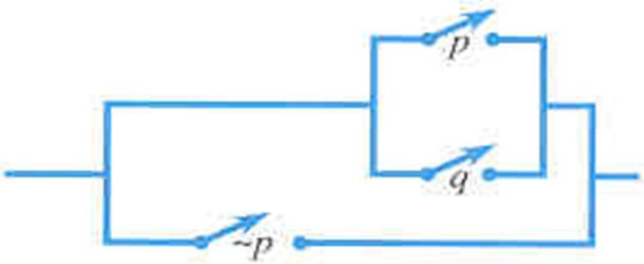
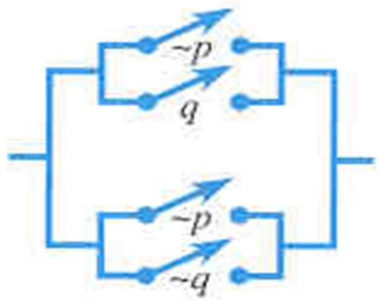


6.



Exercise Set 1.13

2. Draw circuits representing the following statements as they are given. Simplify if possible.

1. $p \wedge (q \vee \sim p)$
2. $(\sim p \wedge \sim q) \wedge \sim r$
3. $(p \vee q) \wedge (\sim p \wedge \sim q)$
4. $(\sim q \wedge \sim p) \vee (\sim p \vee q)$
5. $[(p \vee q) \wedge r] \wedge \sim p$
6. $[(\sim p \wedge \sim r) \vee \sim q] \wedge (\sim p \wedge r)$
7. $\sim q \rightarrow (\sim p \rightarrow q)$
8. $\sim p \rightarrow (\sim p \vee \sim q)$

3. Explain why the following circuit will always have exactly one open switch. What does this circuit simplify to?