**National Commission for Academic Accreditation & Assessment**

**Course Specification**

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| Institution : Majmaah University  |
| College/Department :Zulfi, College of Science |
| Time the Course offered: Each term |

**A Course Identification and General Information**

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| 1. Course title and code: **Ring and Fields MATH 443**  |
| 2. Credit hours : 3h+1h |
| 3. Program(s) in which the course is offered. Bachelor degree of Mathematics |
| 4. Name of faculty member responsible for the course : Rabah Kellil |
| 5. Level/year at which this course is offered : 7th level |
| 6. Pre-requisites for this course : MATH 243 Group Theory**Prerequisites by Topic** 1- Groups and subgroups and their properties to define ring, subrings and ideals2- Quotient sets will be used for Lagrange’s theorem3- factor groups will serve to construct factor rings and the corresponding isomorphism theorems.4- The particular study of the set of integer Z, the set Zn and the set of matrices will give us some examples on different kind of rings (commutative and non commutative ring, invertible element, nilpotent element…). |
| 7. Co-requisites for this course (if any) |
| 8. Location if not on main campus |

**B Objectives**

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| 1. Summary of the main learning outcomes for students enrolled in the course.On successful completion of the module, students should be able to: * Determine the ideals of a given ring
* Determine the factor ring of a ring modulo an ideal.
* Determine the splitting field of a polynomial.
* Characterize prime and maximal ideal in many particular rings.
* To establish that two rings are isomorphic
* Prove that a ring is a field.
* Determine the ideals of a factor ring
* Easily work with polynomials as element of A[X]
* To practice the Euclidian division in K[X] and determine the gcd lcm of polynomials.
* Do calculations inside a finite field.
* Construct finite fields from a field of polynomial over a finite field and an irreducible polynomial.
* Draw the table of Fp[X]/<P(X)>.

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| 2. Briefly describe any plans for developing and improving the course that are being implemented. (eg increased use of IT or web based reference material, changes in content as a result of new research in the field). The course is self contained and doesn’t need to be changed. However the part (Burnside’s theorem, Dihedral groups, Quaternion's groups) can be deleted as it can be developed in a specialised course in Master’s degree studies.  |

**A full academic year is equivalent to 36 Credit hour, which each semester is to be 18 Credit hour. Each course is credited with a number of credit hour (>=2) according to the student's workload (contact hours, laboratory work, examination etc) and accumulation of credits hour is accomplished after successful completion of the course. In this case, one Credit hour is equal 25 – 30 student's workload hour.**

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| **C.Course Description**(Note: General description in the form to be used for the Bulletin or Handbook should be attached)**Contents:** Rings and group of units of a ring. Group of automorphisms of a ring. Ideals and the quotient rings. Principal rings. Prime and Maximal ideals. Field of quotients of an integral domain. Characteristic of a ring. Direct sum of rings. Modules over a ring. Euclidian rings. The ring of polynomials A[X1,X2,…,Xn] over a ring A. Roots of polynomials over a Field K. Extension of fields. Simple and finite extensions of fields. Splitting fields and Algebraic Closures. Finite fields.(The credit point is equal 25-30 hours ) |
| **Topic** | **Contact hours** | **Total of contact hours** | **Self- Study** | **Total hours** |
| **Lecture** | **tutorials** | **Lab** | **Internet** | **Library** | **Homework** | **Discussions** |
| Rings and group of units of a ring, Group of automorphisms of a ring. | 6 | 2 |  | 8 |  |  | 14 |  | 22 |
|  Ideals and the quotient rings. Principal rings. Prime and Maximal ideals. Fields, Field of quotients of an integral domain. Characteristic of a ring | 9 | 3 |  | 12 |  |  | 21 |  | 33 |
| Mid-term 1 | 50mn |  |  |  |  |  |  |  |  |
| Direct sum of rings. Modules over a ring | 6 | 2 |  | 8 |  |  | 14 |  | 22 |
|  Euclidian rings. The ring of polynomials A[X1,X2,…,Xn] over a ring A. Roots of polynomials over a Field K. | 6 | 2 |  | 8 |  |  | 14 |  | 22 |
| Mid-term 2 | 50mn |  |  |  |  |  |  |  |  |
| Finite Fields and Application | 3 | 1 |  | 4 |  |  | 07 |  | 11 |
|  Extension of fields. Simple and finite extensions of fields.  | 6 | 2 |  | 8 |  |  | 14 |  | 22 |
| Splitting fields and Algebraic Closures. Finite fields. | 6 | 2 |  | 8 |  |  | 14 |  | 22 |
| Final Exam | 2H |  |  |  |  |  |  |  |  |
| Total | 45h40mn | 14 |  | 59h 40mn |  |  | 98 |  | 160h40 mn |

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| Country | Equivalence between Systems | For MATH444 |
| KSA | 1 credit hour | 4 CHours |
| Europe | 1.5 ECTS | 6 ECTS |
| USA | 0.625 | 2.5 AC |

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| 4. Development of Learning Outcomes in Domains of Learning For each of the domains of learning shown below indicate:On successful completion of the module, students should be able to: |
| **a. Knowledge** |
| 1. Description of the knowledge to be acquired
* State the axioms defining a ring, Integral domain , invertible element , a field, an ideal, prime and maximal ideals and consequences.
* Deduce simple statements from these axioms.
* Provide examples of different simple ring structures.
* Determine the image and the kernel of a ring homomorphism.
* State, prove and apply some of the classical theorems of elementary Rings and Fields Theory.
* Apply Bezout’s theorem and Gauss’s theorem in Euclidian ring in particular K[X].
* Construct new finite fields in view to be applied to coding and cryptography.
* Study the extension of fields
 |
| 1. Teaching strategies to be used to develop that knowledge

We first introduce new notions, give examples from the simple ones (numbers sets) to those related to matrices, functional sets, we establish the attached properties, we give and prove different theorems related to those notions. Finally we construct new examples and concepts. To well fix the principal facts, weekly homework is usually proposed.  |
| 1. Methods of assessment of knowledge acquired

For the midterm and the final exams (20/100, 20/100, 20/100); in general the methods of assessment are as:* + MCQ on principal theorems
	+ Proving additional notions that can been elaborated from the general study
	+ In general we introduce a short question to control the ability of the student to make the relationship between all the parts that have been studied.

For the homework which represents 20/100 is as stated above.  |
| **b. Cognitive Skills** |
| (i) Description of cognitive skills to be developed* The ability to recognize a ring, ideal and field structure.
* The ability to design new rings by constructing factor ring, to define their ideals and to distinguish the principal, the prime and the maximal ones.
* To have the ability to construct integral domain as a factor of a ring by a prime ideal, and a field as a factor ring of a ring by a maximal ideal.
* The ability to make calculus the ring of polynomials and to be able to determine the gcd of two polynomial and to determine if they are coprime using Bezout’s theorem or any related theorem .
* To be able to apply Gauss’s theorem and in some cases to determine the roots of a polynomial .
* To be able to manipulate the principal ring,
* To be able to construct finite fields as a factor ring of polynomials on finite field by an irreducible ideal.
* To be able to draw the tables of F\_pn[X]/<P>
 |
| (ii) Teaching strategies to be used to develop these cognitive skills* **Explanations and examples given in lectures.**
* **Guidance and supervision of the work developed in tutorial classes.**
* **Allow the students to have brainstorm for any important issues discussed in the class**

 **Site visits** |
| * (iii) Methods of assessment of students cognitive skills
	+ Short questions and discussion during the tutorial class+ short quizzes.
 |
| **c. Interpersonal Skills and Responsibility**  |
| (i) Description of the interpersonal skills and capacity to carry responsibility to be developed |
| (ii) Teaching strategies to be used to develop these skills and abilities |
| (iii) Methods of assessment of students interpersonal skills and capacity to carry responsibility |
| **d. Communication, Information Technology and Numerical Skills**  |
| 1. Description of the skills to be developed in this domain.
	* **The ability to use MAPLE to solve some question relative to roots of polynomials and factoring**
	* **To perform calculus on** F\_pn[X]/<P>
	* **To determine the gcd of two polynomials**
	* **The ability to write a mathematical text in word.**
 |
| 1. Teaching strategies to be used to develop these skills
	* We show that MAPLE can solve many mathematical problems related in particular to ring of polynomials and finite fields and show how MAPLE works.
	* We explore the different sub-packages of the software.
 |
| (iii) Methods of assessment of students numerical and communication skills By easy exercises to solve using computer. |
| **e. Psychomotor Skills (if applicable)****The ability to work on a computer** |
| (i) Description of the psychomotor skills to be developed and the level of performance required**- The ability to work on a computer****- The ability to write a document in some text editor as word.** |
| (ii) Teaching strategies to be used to develop these skills |
|  (iii) Methods of assessment of students psychomotor skills**To control the document produced by the student, the results obtained for the exercises proposed.** |

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| **Course Competency** |
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| **Course Prerequisite Competency** |
| 1. |
| 2. |
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**D- CourseEvaluation methods :**

**[ \*notes: Mid-term1, Mid-term2 and Final Exams are written exam]**

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| **Mark** | **Mid-Exam1**\* | **Mid-Exam2\*** | **Lab.** | **Presentation** | **Homework** | **Project** | **Tutorials** | **Quizzes** | **Peer****project** | **Final Exam\*** | **Total**  |
| **At. Mo. Physics** | 20 | 20 |  |  | 20 |  |  |  |  | 40 | 100 |

Based on your course syllabus, indicate how your course is *assessed*. If an item is given but not assessed, simply leave the last 4 columns blank.

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| **Assessment****Method** | **Number/Type** | **Instructor****Assessed** | **TA/Grader****Assessed** | **Peer/Self****Assessed** |
| Homework | 6To construct and apply the theorems developed in the courseto create new examples from those studied in the courseTo well fix any new notion important to understand the course  |  |  |  |
| Mid Terms/Final Exams | 2 Mid Terms exam and a Final Exams |  |  |  |
| Quizzes |  |  |  |  |
| Individual Projects |  |  |  |  |  |  |  |
| Team Projects |  |  |  |  |  |  |  |
| Lab Assignments |  |  |  |  |
| Computer Assignments |  |  |  |  |
| Computer Tools Used | We use the MAPLE package to **to solve some question relative to roots of polynomials and factoring.** **To perform calculus on** F\_pn[X]/<P>.**To determine the gcd of two polynomials.****The ability to write a mathematical text in word.** |  |  |  |
| Oral Presentations |  |  |  |  |
| Written Reports |  |  |  |  |
| Other |  |  |  |  |

**E.Student Support**

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| 1. Arrangements for availability of teaching staff for individual student consultations and academic advice. (include amount of time teaching staff are expected to be available each week)**Every member of the teaching staff reserves usually 2 hours a day for individual student consultations and academic advising (in the office of the teacher). However the ACADEMIC ADVISING UNIT created recently can be associated to the advising process.** |

##### F. Learning Resources

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| 1. Required Text(s) |
| 1. Essential References
	* Groups, Rings and Fields, J David A.R. Wallace, Springer, 2001,

 ISBN 1540763772-0, 13: 9783540761778* + Introduction to Finite Fields and their Applications, R. Lidl and H. Niederreiter, Cambridge University Press, 1994,

 ISBN 9781139172769, 9780521460941.  |
| 3- Recommended Books and Reference Material (Journals, Reports, etc) (Attach List)Here are a few of my favorite references.Commutative Ring Theory (Cambridge Studies in Advanced Mathematics) , [H. Matsumura](http://www.amazon.com/s/ref%3Dntt_athr_dp_sr_1?_encoding=UTF8&field-author=H.%20Matsumura&ie=UTF8&search-alias=books&sort=relevancerank), [Miles Reid](http://www.amazon.com/s/ref%3Dntt_athr_dp_sr_2?_encoding=UTF8&field-author=Miles%20Reid&ie=UTF8&search-alias=books&sort=relevancerank), June 30, 1989 | ISBN-10: 0521367646 | ISBN-13: 978-0521367646.Commutative Algebra: with a View Toward Algebraic Geometry (Graduate Texts in Mathematics) ,  [David Eisenbud](http://www.amazon.com/David-Eisenbud/e/B001ITROAG/ref%3Dntt_athr_dp_pel_1), March 1, 1999 | ISBN-10: 0387942696 | ISBN-13: 978-0387942698.* + A Guide to Groups, Rings, and Fields , Fernando Q. Gouvêa, Dolciani Mathematical Expositions, 2012, ISBN: 0883853558
 |
| 4-.Electronic Materials, Web Sites etc Use the GAP system that can be download at <http://www.gap-system.org/Releases/index.html> |
| 5- Other learning material such as computer-based programs/CD, professional standards/regulationsSome package of self learning on basic mathematics and editing software as Latex, Scientific work, Lyx |

**G. Facilities Required**

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| Indicate requirements for the course including size of classrooms and laboratories  |
| 1. Accommodation (Lecture rooms, laboratories, etc.)* + There are 20 seats in classrooms and
	+ 20 seats in the computer lab.
 |
| 1. Computing resources
	* 20 seats in the computer lab equipped with MAPLE software
 |
| 3. Other resources (specify --eg. If specific laboratory equipment is required, list requirements or attach list) MATLAB and MATHEMATICA packages are required to equip the mathematics laboratory. |

**H. Course Evaluation and Improvement Processes**

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| 1 Strategies for Obtaining Student Feedback on Effectiveness of Teaching* **Students questioner once during semester**
* **Students department meeting (once during each semester)**
* **Faculty-students periodical meeting (during office hours)**
 |
| 1. Other Strategies for Evaluation of Teaching by the Instructor or by the Department
	1. **Faculty annual evaluation including teaching by the department and the university**
 |
| 3 Processes for Improvement of Teaching* **Attendance of Faculty to workshops offered by Teaching and Learning Development Department**
* **Periodical revision of the method of teaching and the course outcomes**
* **Review of annual course assessment**
 |
| 4. Processes for Verifying Standards of Student Achievement (eg. Check marking by an independent member teaching staff of a sample of student work, periodic exchange and remarking of tests or a sample of assignments with staff at another institution)**Periodical check by the unit of measurement** |
| 5 Describe the planning arrangements for periodically reviewing course effectiveness and planning for improvement.**Department curriculum committee meets in regular basis and recommends revision for improvement. However, we review each semester; in view of the unit of measurement, the methods used in teaching.**  |

**Please fill in this table based on the following criteria:**

1. Based on your course syllabus, provide 3 - 5 *major course objectives* in column 1 along with 2 - 3

*outcomes for each objective* in column 2.

2. In column 3, indicate how the objectives and outcomes in column 1 and 2 map into ME Program Learning Outcomes (PLO)

3. In column 3, indicate how the objectives and outcomes in columns 1 and 2 *map* into the NCAAA Outcomes

4. In column 4, indicate how the objectives and outcomes in columns 1 and 2 *map* into the Asiin criteria

5- Learning outcomes in step 2, 3, 4 are listed in (Program Guidance)

| **Course Objectives:** | **Course Outcomes:** | **PLO** | **NCAAA** | **Asiin** |
| --- | --- | --- | --- | --- |
| 1- Rings and group of units of a ring. Group of automorphisms of a ring. Ideals  | 1- The student should be able to define , identify a ring structure. He also be able to identify its ideals, its prime and maximal ideal.  | a | knowledge | a,b |
| 2- The Group of automorphisms of a ring will have a particular attention.  | c;d | Cognitive | f,j |
| 2- Prime and Maximal ideals. Field of quotients of an integral domain. Characteristic of a ring; Principal rings. | 1- The student should be able to define , identify the invertible elements of a ring. He also be able to construct integral domain and a field from a ring and an ideal. | a,c | Knowledge+Cognitive | a,b,f,j |
| 2- The student should be able to construct the quotient field of a ring and define the elements of the obtained field. | a, f | Knowledge+Cognitive | a,b,f,j;e |
| 3- The student should be able to determine the characteristic of a ring and to use the characteristic to get rise the properties of the ring and solve some equation. He will identify principal ring and the generator of any ideal in principal ring. | a,c | Knowledge+Cognitive | a,b,f,j |
| 3- Direct sum of rings. Modules over a ring.  | 1-Understand the basic definitions concerning the direct sum of rings,- The student will be able to know that the direct sums are also commutative and associative.He also know that If *R* = Π*i*∈*I* *Ri* is a product of rings, then for every *i* in *I* we have a [surjective](http://en.wikipedia.org/wiki/Surjective%22%20%5Co%20%22Surjective)  [ring homomorphism](http://en.wikipedia.org/wiki/Ring_homomorphism)  *pi*: *R* → *Ri*  which projects the product on the *i*th coordinate. He will know that the product *R*, together with the projections *pi*, has the following [universal property](http://en.wikipedia.org/wiki/Universal_property):if *S* is any ring and *fi*: *S* → *Ri* is a ring homomorphism for every *i* in *I*, then there exists *precisely one* ring homomorphism  *f*: *S* → *R* such that *pi* ∘ *f* = *fi* for every *i* in *I*. | a,c | Knowledge+Cognitive | a,b,f,j,e |
| 2- Understand the basic definitions concerning modules;* Understand homomorphisms and isomorphisms of modules and be able to construct examples;
* Be able to describe submodules in terms of systems of generators;
* with quotient modules;
* Understand and be able to work with localized modules;
* Understand the notions of k-algebras and homomorphisms of k-algebras;
 | a,c | Knowledge+Cognitive | a,b,f,j,e |
| a,c | Knowledge+Cognitive | a,b,f,j,e |
| 4- Euclidian rings. The ring of polynomials A[X1,X2,…,Xn] over a ring A. Roots of polynomials over a Field K.  | 1- The student should be able to define an Euclidian division in the ring. The specific case of the ring of polynomials on a field will have a particular attention.  | c,d | Knowledge+Cognitive | f,j,e,h |
| 2-The student will be able to perform all the operations related to the arithmetic of the polynomials as Bezout’s and Gauss’s theorems, the gcd... | c,d | Knowledge+Cognitive | f,j,e,h |
| 3-The roots of a polynomial will take an important attention and the student will be able to factor a polynomial and construct ideals generated by ideal to prepare to Hilbert theorem. | c,d | Knowledge+Cognitive | f,j,e,h |
| 5- Extension of fields. Simple and finite extensions of fields. Splitting fields and Algebraic Closures.  | * 1- the student will use the subfield criterion to determine whether a given subset of a field is a subfield;
* and understand the distinction between algebraic and transcendental elements of a field extension of a base field;
* use Eisenstein's irreducibility criterion for polynomials;
* use the minimal polynomial of an algebraic element α of a field extension of a base field F to express the inverse of a non-zero element of F(α) in the from p(α) for some polynomial p with coefficients in F;
 | c,d | Cognitive | f,j,e,h |
| * 2- He will work with primitive n-th roots of unity and the cyclotomic polynomial φn; and prove that φp is irreducible when p is a prime number; he will understand what is meant by a constructible point of the Euclidean plane and a constructible real number;know and be able to use a field-theoretic criterion for determining whether a given point of the Euclidean plane is constructible.
* He will understand the concept of the group of automorphisms of a field extension.
 | c,d | Knowledge+Cognitive | f,j,e,h |
| Finite fields. | * 1- be able to describe all the finite fields and their groups of automorphisms;
* understand the concept of the algebraic closure of a finite field.
 | c,d | Knowledge+Cognitive | f,j,e,h |
| 2-Construct finite fields from F\_p[X] and an irreducible polynomial. | c,d | Cognitive | f,j,e,h |

**Prepared by :Dr Rabah Kellil**

**Date of Preparation: 12-04-2014**

**Revised:**

**Head of Department:**