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Pareto Optimal Path Generation Algorithm in Stochastic Transportation Networks

MAHMOUD OWAIS[1,2] AND ABDULLAH ALSHEHRI[3]

[1]Department of Civil Engineering, Faculty of Engineering, Assiut University, Assiut 71515, Egypt
[2]*(on leave)* Department of Civil Engineering, Faculty of Engineering, Majmaah University, Majmaah 11952, Saudi Arabia
[3]Department of Civil and Environmental Engineering, Faculty of Engineering, Majmaah University, Majmaah 11952, Saudi Arabia

Corresponding author: Mahmoud Owais (m.owais@mu.edu.sa)

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ABSTRACT Routing problems play a crucial part in urban transportation network operation and management. This study addresses the problem of finding a set of non-dominated shortest paths in stochastic transportation networks. Instead of the previous practice of assuming the travel time variability to be tracked by a known probability density function, it is extracted from the existing correlation between the traffic flow and the corresponding links' time. The time horizon is divided into time intervals/slots in which the network is assumed to experience a static traffic equilibrium with different traffic conditions for each slot. Starting with Priori demand information, prior generated paths, and a chosen traffic assignment method, the proposed methodology conducts successive simulations to the network intervals. It manages to draw both links and paths probability distribution of their travel time considering the correlation among them. Then, multi-objective analysis is conducted on the generated paths to produce the Pareto-optimal set for each demand node pair in the network. Numerical studies are conducted to show the methodology efficiency and generality for any network. The expected travel time and the reliability could be drawn for each path in the network.

INDEX TERMS Path generation, Pareto optimal, stochastic transportation network, traffic assignment, demand simulation.

I. INTRODUCTION

In transportation networks (TN), finding the best route/path between an origin/destination pair (O/D) is always a crucial task for both operators and users. Usually, operators study this type of problem to run the network efficiently. Operators do not bother themselves to transport each user in the shortest path, but alternatively, they ensure the overall harmony of the network. This appears apparently in the design of transit routes as found in [1]–[3]. On the other side, users need the generated routes to be the fastest way to transport them from their origins to the desired destinations. Regarding the two perspectives, researches have split in the methodologies of creating the route choice set for each O/D pair i.e. user perspectives or operator ones. This confliction is apparent in [4] work where generated routes are deviated from the shortest path to alleviate the network congestion while trying to maintain a level of acceptance to the users.

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Also, the routing problem in TN differs when the generated routes are tracking the perception of the network users. The shortest path is not always the choice of most users due to the lack of familiarity/information of the network. Therefore, the problem turns into a route choice problem as in traffic assignment approaches, and thus, there are different methods of generating the routes based on the selected traffic assignment method [5]–[7]. On the contrary, the real-time shortest path is the target to be produced upon one's request (e.g., google maps).

Another dimension of the complexity is the TN conditions representation and data. Many levels of network representation are found in the literature which vary from simple to complex. In the route-finding algorithms, the description of arcs/links travel time is the most crucial factor. When travel time on arcs is considered deterministic (in-variant and known for sure), the shortest path is easily found by the Dijkstra algorithm [8]. However, in the real-world, each link travel time is variant and hard to predict. It depends on the level of traffic on it, so the algorithm fails to find the optimum path until the traffic is observed on all links in real-time which

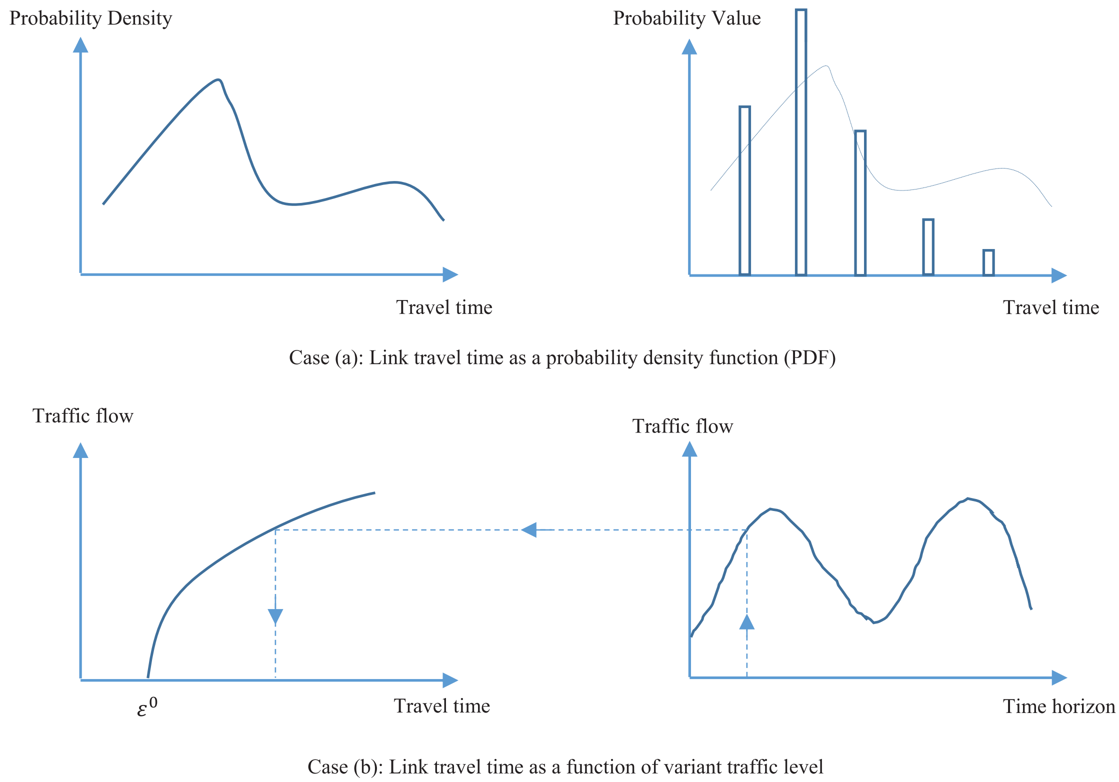


FIGURE 1. Different realizations of link travel time.

unpractical premise [9]–[11]. The traffic is also stochastic with time causing travel time uncertainty turning the TN to what is called Stochastic Transportation Network (STN).

The problem of travel time stochasticity is usually simplified by assuming the existence of probability density function (PDF) for each link. Then, the expected travel time on each arc could be used with any of the deterministic shortest path techniques [12]. This stochasticity/variability is thought in two ways; as stationary or non-stationary random variables [13]. The difference between the two approaches is the PDF consideration for each arc time to change with time or not. In Fig. 1, we could track the different perspective of tackling the travel time stochasticity. Case (a) illustrates the assumption that the travel time has a continuous PDF, which could be simplified by discretizing the time values with respect to the probability values to be summed up to 1. Case (a) represents the common assumption in most of the work done regarding the stochastic routing problem. However, case (b) reflects our realization of the problem. It depicts the real correlation between traffic variability and travel time that has a minimum value (ε^0) resulted from the free flow speed which is able to increase with the various traffic conditions.

In this study, the problem of vehicle routing in STN is tackled to generate a set of paths for each node pair (O/D). In the set, each path might be the shortest path in this variable environment. The path sets are generated under users' perspective while considering this routing does not affect the network conditions. An algorithm based on successive simulations to the STN is developed to provide the set of non-dominated paths (Pareto-optimal) as none of them can be eliminated from the choice set when the trip is about to start. Also, a priori probability distribution could be drawn to examine the reliability of the selected route for a given travel time budget.

The structure of this article is as follows. Section 2 draws a concise state of the art for the problem, whereas section 3 gives the basic adopted concepts. The algorithms developed in the solution method are reported in section 4. In Section 5, real case networks are used to evaluate the methodology. Section 6 presents the conclusion.

II. STATE OF THE ART

In this part, a quick review is made on how the best path generation problem is dealt with in the literature and what the common assumptions are taken to solve the problem. Then, we show our relatively new direction to alleviate the mathematical complexity of the problem and to approach the reality of traffic conditions in TN.

The route generating problem is the main problem in numerous applications of different fields, such as communications, electrical networks, and transportation networks [14]. The shortest path in deterministic networkers is the core of all routing algorithms. One of the dilemmas added to the problem is considering travel times are time-dependent, and the delay occurred on the links are caused by known functions. The shortest path labeling algorithm is modified to contain this new part of complexity [15]. This assumption is still unsuitable to TN because finding these types of fixed time-dependent functions are impossible in the real world. Reference [16] adds a dynamic side to the problem considering conditional probability between the arcs time and the environment. A Markovian representation helps to find an optimal policy at each node of moving forward in the considered arc or waiting until the conditions improve. Although the network horizon is a continuous interval, discretizing it into time slots is regarded as the best way to alleviate the problem complexity [17]. The slots are connected with a time-dependent function that gives each link travel time according to the arrival time to the link's start node. It may be considered an extension to the conventional deterministic shortest path problem, which can be solved with recursive Bellman's [18] principle of optimality. However, in the case of random travel times, Bellman's equation does not hold.

Therefore, the deterministic time-dependent assumption becomes useless in networks such as TN, where there is a raise to the stochasticity (uncertainty) in travel times. This uncertainty stems from the traffic incidents, lack of information or demand variability [19]. It makes the problem not only minimizing the generated paths travel times but also increasing the reliability of the paths against such uncertainties. Reference [20] is the pioneering work to focus on finding the shortest path on networks with time-dependent random travel times. A notion of finding the least expected travel time (LET) path is solved with iterating branch and bound technique. The formulated problem could be generalized to the hyper-path issues as seen in transit assignment problems when considering the adaptive route choice. Searching for the LET paths is extended in [13] with label correcting algorithms. However, prohibiting paths with cycles is not guaranteed without maintaining First in – First out (FIFO) property which is not a realistic assumption in the TN routine. FIFO states for any arc $(i-j)$, you cannot reach (j) earlier than anyone has already left node (i) [21]. This property eliminates the fact of a car overtaking in the TN or that you can improve your travel time by moving later [16].

There is a common belief that adaptive routing can always enhance a trip travel time. In the STN, LET paths are determined before conducting the trip using the expectation of travel times as random variables. After that, some information about the upcoming links travel time might be revealed during the journey (en-route). Then, a rerouting process could be done based on this information [22], [23]. In case of the travel times on links are assumed to be conditioned to the arrival time to the link's start node, the adaptive routing is used when the actual arrival time is revealed en-route [24].

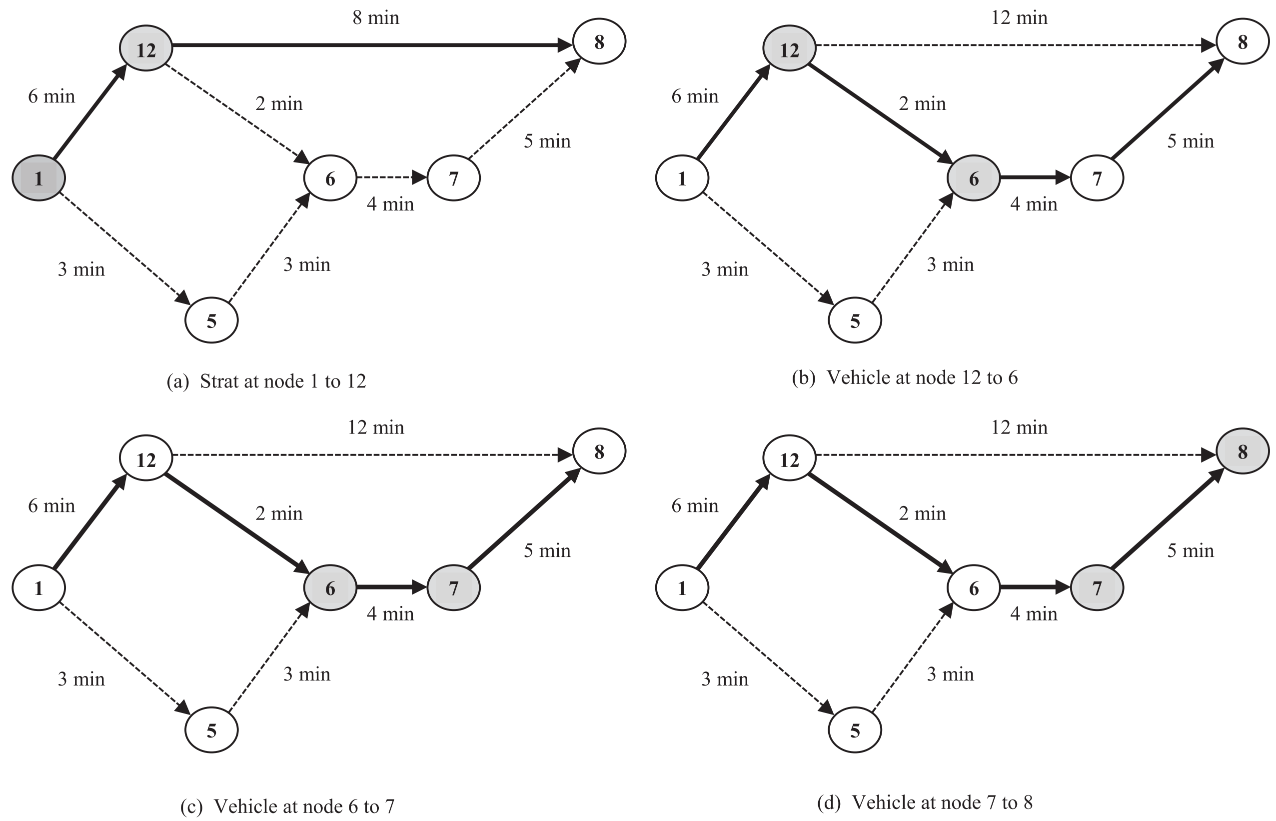
Dealing with the problem in its generality without specifying it as a TN problem makes many studies ignore the correlation among links travel time [25]. To tackle this property in an analytical process, some studies assumed the existence of the correlation due to bad weather or traffic accidents. So, they confined it to the next link status in a local way [26], [27]. Even the studies, which considered the extended correlation among all TN links, assumed the existence of discrete link times joint distribution or a covariance matrix. However, they did not specify a logical way to obtain this information in the real-world [28]–[31]. Drawing different samples for different travel time realizations using Monte Carlo (MC) simulation was their best approach to drive the path times distribution from the predefined links correlation. In the independent link travel time assumption, the simulation is not a requirement [32].

A few studies focused on the multi-objective nature of the problem in which the expected path time is not the only criterion for the choice [33]–[35]. Other criteria, such as path's time-span of variation and its reliability, could be considered.

The travel time correlation stems mainly from the complex interrelation between the demand on a network O/D pairs, the available route choice set, and links travel time flow dependent. Fortunately, traffic assignment models could handle this correlation efficiently and produce the links' time covariance matrix [36], [37].

The computational burden constitutes the main challenge for most routing algorithms since the paths are requested on time by users. Existing routing formulations are NP-hard even under restrictive assumptions. So, off-line planning might be an excellent solution to be examined for the problem. Starting with prior constructed routes and then storing them for on-line use is possible with new advances in computer servers. For the planning part, the expected network conditions could be reflected by successive demand simulation [3].

To this end, we could highlight some gaps existed in the literature and the potential contribution of this work. The correlation among links travel time has received little attention by either ignoring or treating locally. Even in the studies where link times are assumed to be random variables with known joint distributions, there is no guide on how to acquire this information. Alternatively, the proposed methodology tries to extract these latent pieces of information by mimicking the TN understudy. Although the real demand for a network is unknown, it fluctuates around the available O/D matrix (historical or estimated) according to a level of expected variation. Based on this assumption, synthetic demand data could be created for a given time horizon which is divided into time intervals/slots. The TN is assumed to work under equilibrium for each time slot in which travel times on links are constant but unknown for certain. Therefore, the stochasticity would come from loading the randomly generated O/D matrix that varies in each time slot due to the normal fluctuation of demand over time. Paths travel time probability distribution and correlations among their links are drawn numerically from different O/D samples and the adopted traffic loading method. After several realizations to the TN, candidate routes for each O/D pair are stored for the multi-objective analysis to produce the final set. Thus, both the time correlation and probability distribution densities for links and paths are



(a) Strat at node 1 to 12

(b) Vehicle at node 12 to 6

(c) Vehicle at node 6 to 7

(d) Vehicle at node 7 to 8

FIGURE 2. Simple network with variable travel times on arcs through four realizations.

tackled inherently without the need of pre-specification. In Short, combining multi-objective pathfinding in the STN and taking into consideration the link travel time spatial correlations are the notable contributions of this work.

III. PROBLEM STATEMENT

The problem is clearly viewed as how to find the best path/paths connecting an O/D pair in a time-varying transporting network. The prior routes selection is the promoting approach in this study. It is the process of constructing the paths in a network before routing requests (off-line planning). To justify this decision, we start with the following proposition:

Proposition: "The adaptive routing technique does not always lead to the optimal path in the STN."

Proof: Let us track a routing process in a simple network given in Fig. 2. It is required to travel from the node (1) to (8) with the adaptive routing approach. The network travel times are revealed en-route, then the routing decision could be adapted accordingly. The network consists of two distinct main paths (1-12-8) & (1-5-6-7-8) with the possibility to transfer from the first path to the second through the link (12-6). In the start of routing, case (a), the shortest route is (1-12-8), with total travel time 14 min. When the vehicle reaches node (12), the decision is changed based on the revealed information to move to the node (6) to avoid link (12-8) saving one minute. The trip ends at node (8) with total travel time 17 min following (1-12-6-7-8), whereas path (1-5-6-7-8) achieves 15 minutes during the four-time realizations of the network. Therefore, it is concluded that adaptive routing is not always the best solution, and hence determining a prior route to follow may lead to a better path. □

A. THE STOCHASTIC TRANSPORTATION NETWORK

In this study, the STN is represented with a graph $G = (N, A, \hat{W}, \Theta, \zeta)$, where N is the set of nodes $N = \{1,2,..,i,..,j..,k..,l,..,\hat{\eta}\}$ which are connected by directed arcs/links $A = \{(i,j): i,j \in N\}$. The demand node pairs with start (s) and end (r) are grouped in the vector $\hat{W} = \{1,2,\ldots,w,\ldots,\hat{w}\}^t$, the superscript t refers to the transpose. The analysis time horizon Θ is discretized in the set $\Gamma = \{t_0,\ t_0+\delta,\ t_0+2\delta,\ldots,\tau,\ldots,\ t_0+n\delta\}$ with time interval δ in which the network experience a temporary static equilibrium resulted from demand vector $D_\tau = \{d_\tau^w :|\ D_\tau\ |=|\hat{W}| = \hat{w}\}$ which is correlated with the selected time slot τ. ζ is the set of links travel time $\{\ldots,\varepsilon_{ij}^{\tau},\ldots\}$ resulted from assigning D_τ to the network. To this end, the network representation is not stochastic since the travel time could be calculated directly after assigning the demand vector. In practice, D_τ is not known for certain, so dealing with it as a random variable brings back the stochasticity to our representation. The demand uncertainty turns link times to the stochasticity which

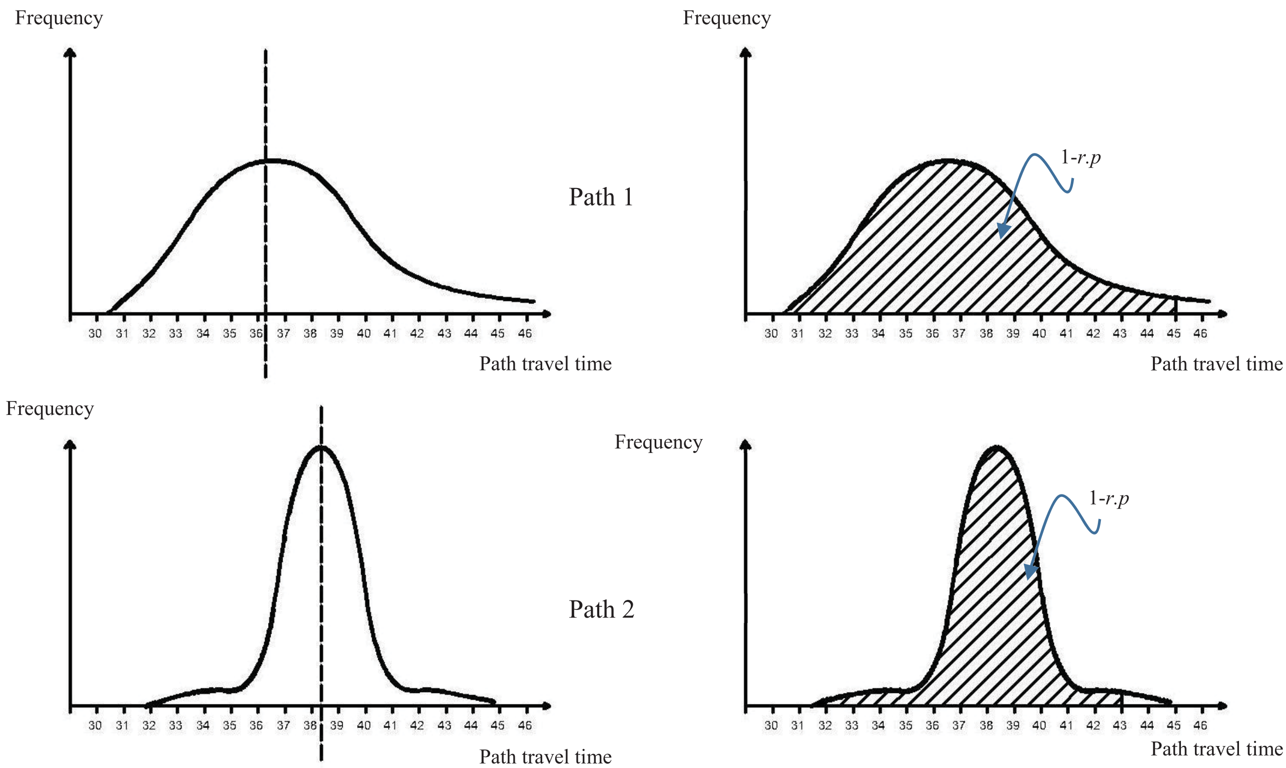


FIGURE 3. Two non-dominated paths with travel time probability distribution.

can be characterized by the covariance matrix $\sum = \{\sigma_{ij,w}\}$. Each node pair w is assumed to be connected by a number of paths $H_w = \{h_\Lambda^w : |\Lambda| = \bar{\Lambda}, H_w \subset H\}$ where Λ is the path distinguishing number and H is the group of all network's paths. The relations among the network's elements are summarized as follows:

$$\varepsilon_{ij}^{\tau}(f_{ij}^{\tau}) = \varepsilon_{ij}^{0}\left[1+\lambda_{ij}\left(\frac{f_{ij}^{\tau}}{Q_{ij}}\right)^{\chi_{ij}}\right] \quad \forall (i,j) \in A \tag{1}$$

$$T_{\tau}^{h_{\Lambda}^{w}} = \sum\nolimits_{ij} \varepsilon_{ij}^{\tau}\delta_{ij}^{h_{\Lambda}^{w}} \forall w \in \hat{W}, \quad \forall h_{\Lambda}^{w} \in H_w \tag{2}$$

$$f_{ij}^{\tau} = \sum\nolimits_{w}\sum\nolimits_{\Lambda} F_{h_{\Lambda}^{w}}^{\tau}\delta_{ij}^{h_{\Lambda}^{w}}, \quad \forall (i,j) \in A \tag{3}$$

$$F_{h_{\Lambda}^{w}}^{\tau} = d_{\tau}^{w} S(T_{\tau}^{h_1^w}, T_{\tau}^{h_2^w}, \ldots, T_{\tau}^{h_{\Lambda}^{w}}, \ldots, T_{\tau}^{h_{\bar{\Lambda}}^{w}}) \tag{4}$$

In eq. (1), the link travel time ε_{ij}^{τ} is functioned of the link flow f_{ij}^{τ} at time step τ through the well-known Bureau of Public Roads (BPR) formula [38], where ε_{ij}^{0} is the link travel time under free flow condition, f_{ij}^{τ} and Q_{ij} are the link flow and capacity respectively. λ and χ are factors that determine how the link flow affects its time. In eq. (2), for any path h_{Λ}^{w} at time step τ the total travel time T is calculated by summing the times of links comprising the path where δ is the incident factor equals 1 when the link $(i–j)$ is on the path, 0 otherwise. In eq. (3), the link flow f_{ij}^{τ} results from the paths flow F that passes through it. The path flow is a result of splitting the demand d_{τ}^{w} among the different paths connecting the O/D pair w through the split share S which is a function of all the paths times. It is worth noting that the mapping function S comes from the traffic assignment step, as will be shown later. It is evident from (1) to (4) that the link travel time is inherently a function of the demand vector. Unfortunately, knowing/assuming a PDF for D_{τ} does not help deduce the link travel times analytically. Therefore, simulation is the best way to track links travel time variability without the need for monitoring the complex intertwined relationships.

B. PATHS EVALUATION AND COMPARISON

In deterministic networks, finding the optimal path is an easy task considering the static link attributes of optimality like travel time or general cost. In the STN, the problem is more complicated. Each path has an ideal/primary total travel time T_0 comes from summing its links ε_{ij}^{0} which can be dealt with as a lower bound for that path. This lower bound is unlikely experienced by the users due to the links time increase with the flow, as in eq. (1). The flow fluctuation and uncertainty turn the path travel time to a random variable with unknown PDF. Hence, the path optimality cannot be measured by achieving the least T_0. However, it could be an indicator of how efficient the path is, but it is not the only judge. To look more in-depth, let us imagine the time of two paths depicted in Fig. 3, which both follow normal distributions (this assumption is only for the illustration purpose) and connect the same O/D pair w. Although path 1 is shorter than path 2 considering the free-flow travel time T_0, stochastically none of them is better than the other. In other words, the two are non-dominated paths (Pareto-optimal). The non-dominance results from the overlap of the two distributions that each path may be the shortest with probability > 0. Another two reliability measures may be considered, namely; α, β. The former is the expected path time (e.g., the mean of path 1 = 36.2min & path 2 = 38.3min) whereas the latter is the worst possible time given a risk probability ($r.p$) value (e.g., with risk = 5%, path 1 time $\leq$ 45 min & path 2 time $\leq$ 43 min). It is worth noting that if $r.p$ the threshold is increased to a particular value, path1 will dominate path 2. Mathematically, path evaluation criteria are represented as follows;

$$Z_1 = \int_{T_0}^{\infty} Tp(T)\,dT \tag{5}$$

$$Z_2 = P^{-1}(1 - r.p), \quad \because\ P = \int_{T_0}^{z_2} p(T)\,dT \tag{6}$$

In eq. (5), the expected travel time is calculated from the path's PDF, whereas eq.(6) gives the time, which ensures that the user would not be delayed over with probability = $r.p$. The two criteria could be used to judge each path performance in the STN. The existence of more than one objective raises the term of multi-objective analysis. It is concerned with conducting an exclusive analysis to produce the Pareto-optimal set regarding $Z_1 \& Z_2$.

As mentioned before, in reality, the paths time does not follow a known PDF; besides that, their probability distribution varies with the time. Therefore, $Z_1 \& Z_2$ are examined to be estimated numerically, as will be shown later in the methodology section.

C. THE STOCHASTIC DEMAND

The stochastic demand is the core assumption in the proposed methodology since it is assumed that the demand change controls all the variability in the STN. Provided that the total demand (U_τ) in the network during a time slot is a random variable with known expected mean μ_τ and standard deviation σ_τ and there is a reference demand vector $D^0 = \{d_0^1, d_0^2, \ldots, d_0^w, \ldots, d_0^{\hat{w}}\}c$. The existence of such information is a common assumption in the literature [37], [39]–[45]. Then, the D_τ is expressed as follows;

$$D_\tau = U_\tau K + D_\eta \tag{7}$$

$$K = \frac{1}{\sum_w d_0^w} D^0 \tag{8}$$

where; $\boldsymbol{K}$ is a vector of size ŵ and with $\boldsymbol{k_w}$ values which are positive real constants measuring the relative weight of the O/D pair $\boldsymbol{w}$ flow with respect to the total traffic flow $\boldsymbol{U_\tau}$ at a time step $\boldsymbol{\tau}$. $\boldsymbol{D_\eta}$ is also a vector of size ŵ and its values of $\boldsymbol{\eta_w}$ are random variables with null mean and $\boldsymbol{\gamma_w}$ standard deviation. $\boldsymbol{\eta_w}$ is added to represent the independent part of demand fluctuation. It is obvious now that the $\boldsymbol{D_\tau}$ elements are correlated random variables due to the share at $\boldsymbol{U_\tau}$.

IV. METHODOLOGY

In this section, the methodology is designed to be generally applicable to the pathfinding problem in the STN. It considers the temporal travel time correlations in the network besides the absence of information about link travel time PDF. Given that problem, it is required to produce a set of non-dominated paths for each O/D pair with a priori probability distribution. Three basic models are incorporated to formulate the methodology, namely; prior path generation, traffic loading stage, and multi-objective analysis. In the next subsections, the models' implementation is illustrated. Then, the connection among them is drawn in the final algorithm presentation.

A. PRIOR PATH GENERATION ALGORITHM

In this step, a path set is generated for each O/D pair as a pool of selection. These paths are considered as the population that contains the aimed solution set. They should reflect the diversity and potentiality of finding suitable alternatives between each demand node pair. Many algorithms are found in the literature to create these paths. Shortest path and k-shortest path are the commonly used one [46] which could be calculated with ε_{ij}^0 that represents link topological shortness as it is the time required to pass the link with no traffic considered. The set of the k-shortest path is as follows:

$$\begin{aligned} H_w &= \{h_1^w, h_2^w, \ldots, h_\Lambda^w\} \\ &= min\{\textstyle\sum_{ij} \varepsilon_{ij}^0 \delta_{ij}^{h_1^w}, \sum_{ij} \varepsilon_{ij}^0 \delta_{ij}^{h_2^w}, \ldots, \sum_{ij} \varepsilon_{ij}^0 \delta_{ij}^{h_\Lambda^w}\} \end{aligned} \tag{9}$$

Eq. (9) gives the required path set for each O/D pair w where paths are generated in ascending T_0. Also, paths that exceed a certain value ρ (circuitous factor) from the shortest path are not allowed as in the following inequality:

$$\frac{\sum_{ij} \varepsilon_{ij}^0 \delta_{ij}^{h_\Lambda^w}}{min \sum_{ij} \varepsilon_{ij}^0 \delta_{ij}^{h_\Lambda^w}} < \rho, \quad \forall h_\Lambda^w \in H_w \tag{10}$$

For example, transportation planners commonly regard paths with $\rho = 1.5$ as circuitous and not favorite for routing [47].

In this study, Yen's k-shortest path algorithm is modified to reflect our vision of the routing solution. The pseudo-code of the route generation algorithm is as follows:

Precondition: *connected street network with* ε_{ij}^0 *on links,* ρ *factor,*

Postcondition: $H_w \in H$

a. *For each* $w(s, r) \in \hat{W}$ *do*
   i. *Open an empty set* H_w
   ii. *Set* $\Lambda = 1$
   iii. *Determine the shortest path from s to r (Dijkstra-algorithm)*
   iv. *Mark route links in its sequence as* $g^\Lambda = (1^\Lambda, 2^\Lambda, \ldots\ldots \widehat{a^\Lambda})$
   v. *Check* ρ *as in eq. (10) if ok then add* h_r *to* H_w*, otherwise, go (vii)*
   vi. *Delete all* g^Λ *links, then check* (s, r) *connectivity, if connected then set* $\Lambda := \Lambda + 1$ *and go (iii), otherwise go next*

vii. *set $q = \Lambda$ as none overlapped routes*
viii. *For $\Lambda = 1$ to q*
ix. *Rerun all deleted links*
x. *Delete all route g^{Λ} links*
xi. *return g^{Λ} next link and force it in the new route set ($\bar{\Lambda}: = q + 1$)*
xii. *check (s, r) connectivity, if connected determine the shortest path from r to s, otherwise go (xi)*
xiii. *Check ρ if ok then add h_{Λ} to H_w, otherwise, go (xi)*
xiv. *Set new $q = \bar{\Lambda}$ as overlapped routes*
xv. *if g^{Λ} links reached a^{Λ} go next, otherwise go (xi)*
xvi. *do the same procedure backward, then go next*
xvii. *End for*
xviii. *add H_w to H*

b. *End for*
c. *Return H*

In steps (*i*) to (*vii*), the algorithm tries to generate k-shortest paths that do not overlap in any route section. For each demand node pair *w*, there would be *q* independent paths. Whereas steps (*viii*) to (*xvii*) use the initial set *q* to generate a number of overlapped paths where the overlapping is permitted gradually from one link to almost all the path links (i.e., all next generated paths are based in perturbation of the initial group). The algorithm cut route branching when ρ exceeds the predefined threshold. This could avoid the need for storing unnecessary paths and give the required variation for the selection stage (multi-objective analysis stage).

B. TRAFFIC ASSIGNMENT

Traffic assignment is one of the four major conventional travel demand forecasting models which has been studied for several years. The product of the assignment is the link flows that constitutes essential information for traffic planners at both strategic planning and operational aspects of urban networks. Traffic assignment is not a straight forward problem that does not have a single closed-form solution algorithm due to its different interrelated components with different perspectives. The differentiation among the solution methods begins with the level of network representation, demand centers determination and Origin/Destination (O/D) matrix estimation to end up with the assumed route choice model, the expected type of equilibria through the network and the mathematical approach to convergence at a solution.

The selection of a traffic assignment method depends on which one is believed to truly resemble the people's movement and route choices in the network under study. In this study, the deterministic user equilibrium (DUE) assignment model is chosen for the analysis noting that any traffic assignment model suits the proposed framework. All assignment models share the same input and output but differentiate in behavior representation, parameters to calibrate and solution method. DUE is easy to be solved without many parameters to calibrate, so it alleviates the burden of the notions in this article. It also has a non-path based solution algorithm which makes it a non-biased method to evaluate the generated paths. DUE is formulated as follows:

$$\min \sum_{ij} \int_0^{f_{ij}^{\tau}} \varepsilon_{ij}^0 \left[1 + \lambda_{ij} \left(\frac{f_{ij}^{\tau}}{Q_{ij}}\right)^{\chi_{ij}}\right] df \tag{11}$$

$$s.t\; f_{ij}^{\tau} = \sum_{w} \sum_{\Lambda} F_{h_{\Lambda}^{w}}^{\tau} \delta_{ij}^{h_{\Lambda}^{w}}, \quad \forall (i,j) \in A \tag{12}$$

$$d_{\tau}^{w} = \sum_{\Lambda} F_{h_{\Lambda}^{w}}^{\tau}, \quad \forall w \in W \tag{13}$$

$$F_{h_{\Lambda}^{w}}^{\tau} \geq 0, \quad \forall h_{\Lambda}^{w} \in H, \; \forall \tau \in \Gamma \tag{14}$$

To solve this set of equations for any time slot τ, the convex combination method is adapted as follows:

Precondition: *connected street network, non-empty D_{τ}*
Postcondition: *link flows $\left(f_{ij}^{\tau}\right)$ – link travel times ε_{ij}^{τ}*

Step 0: assign each flow d_{τ}^{w} to the shortest path ($\min \sum_{ij} \varepsilon_{ij}^0 \delta_{ij}^{h_{\Lambda}^{w}}$). These yields flow on links (x_{ij}^{ctr}), set the counter (ctr=1).

Step 1: update $\varepsilon_{ij}^{\tau}(x_{ij}^{ctr}) = \varepsilon_{ij}^0 \left[1 + \lambda_{ij} \left(\frac{x_{ij}^{ctr}}{Q_{ij}}\right)^{\chi_{ij}}\right]$

Step 2: assign each flow d_{τ}^{w} to the shortest path ($\min \sum_{ij} \varepsilon_{ij}^{\tau}(x_{ij}^{ctr}) \delta_{ij}^{h_{\Lambda}^{w}}$). This yields a set of (auxiliary) flows (y_{ij}^{ctr}).

Step 3: find $\pi^{ctr}(0 \leq \pi \leq 1)$ that solves $\min \sum_{ij} \int_0^{x_{ij}^{ctr} + \pi(y_{ij}^{ctr} - x_{ij}^{ctr})} \varepsilon_{ij}^{\tau}(f) df$

Step 4: set $x_{ij}^{ctr+1} = x_{ij}^{ctr} + \pi^{ctr}(y_{ij}^{ctr} - x_{ij}^{ctr})$, $\forall (i,j) \in A$

Step 5: if $\frac{\sqrt{\sum_{ij} (x_{ij}^{ctr+1} - x_{ij}^{ctr})^2}}{\sum_{ij} x_{ij}^{ctr}} \leq \kappa$, set $x_{ij}^{ctr+1} = f_{ij}^{\tau}$ then stop, otherwise set ctr: = ctr+1 and go step 1.

C. MULTI-OBJECTIVE ANALYSIS

In this stage, the generated routes are ranked and filtered under the multi-objective analysis that examines each solution's dominances. At first, the selected objectives Z_1 and Z_2 need to be estimated for the generated paths, then the multi-objective analysis could be conducted to produce the set of non-dominated paths. The complete algorithm of the methodology in which all the parts mentioned above are connected for that purpose is as follows;

Input: network structure – link attributes ($\varepsilon_{ij}^0, \lambda_{ij}, \chi_{ij}, Q_{ij}$) – μ_{τ} – $\sigma\tau$ – D^0 – K, D_{η} – convergence tolerance (κ), No of maximum iterations ($itr._{max}$) – ρ – ψ_{max}, Θ, Γ, (1–r.p)
Precondition: *connected street network, non-empty D_{τ}*
Postcondition: *Pareto optimal path set ($\hat{C}$)*

1. *Perform path generation Algorithm 1 to obtain the $H = \{H_1, H_2, \ldots\ldots, H_{\hat{w}}\}$.*
2. *set itr. = 1*
3. *for $\tau = t_0$ to $t_0 + n\delta$ do*
    4. *generate U_{τ} randomly according to μ_{τ} & σ_{τ} and assumed probability distribution*
    5. *estimate D_{τ} according to eq. (7&8)*
    6. *perform DUE assignment according to step.s 0 to 5 as in Algorithm 2.*

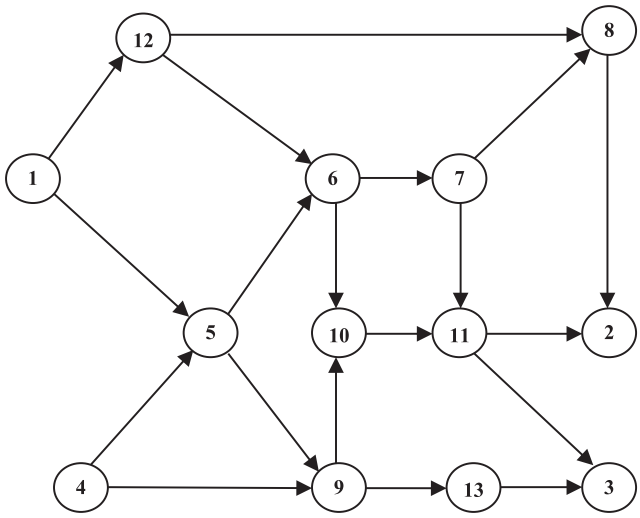


FIGURE 4. The Nguyen–Dupuis network structure: nodes and links.

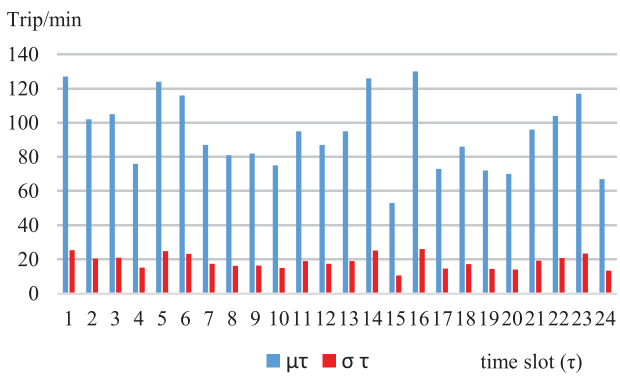


FIGURE 5. Total demand mean and standard deviation variation for the Nguyen–Dupuis network.

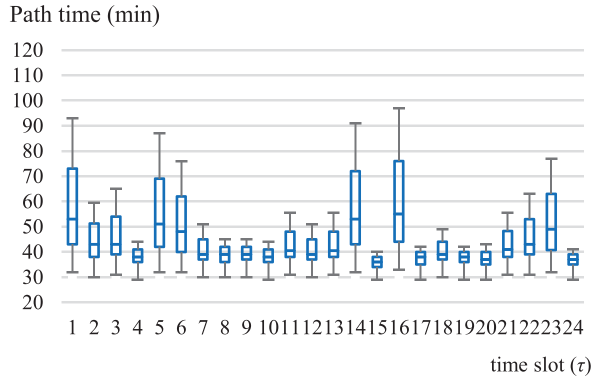


FIGURE 6. Box Plot for path 1 time variations through the different time slots.

TABLE 1. Parameters of the Nguyen–Dupuis network.

Link (i-j)	ε_{ij}^{0}	Q_{ij}	λ_{ij}	χ_{ij}
1-5	7	35	1	4
1-12	9	56	1	4
4-5	9	56	1	4
4-9	12	75	1	4
5-6	3	14	1	4
5-9	9	42	1	4
6-7	5	28	1	4
6-10	5	28	1	4
7-8	5	28	1	4
7-11	9	70	1	4
8-2	9	70	1	4
9-10	10	56	1	4
9-13	9	56	1	4
10-11	6	54	1	4
11-2	9	56	1	4
11-3	8	56	1	4
12-6	7	14	1	4
12-8	14	70	1	4
13-3	11	56	1	4

7. *estimate ε^{τ} for each link (ij) in A according to eq.(1), then save it as $\varepsilon_{itr.}^{\tau}$.*
8. *calculate T_{τ} for each path $h \in H$ according to eq.(2), then add $T_{\tau}^{itr.}$ to $\widehat{T_h}$ set*

9. *end for*
10. *if $itr. = itr._{max}$, go next, otherwise set $itr. := itr+1$, then go to 2.*
11. *for each path $h \in H$*
12. *sort all path times in $\hat{T}_h$ ascendingly according to $T_{\tau}^{itr.}$*
13. *calculate $Z_1 = \frac{\sum_{itr.}\sum_{\tau} T_{\tau}^{itr.}}{itr._{max}+n+1}$*
14. *determine $Z_2 =$ the time of the $(1\text{-}r.p)(itr._{max}+n+1)$ path number in the sorted order $\widehat{T_h}$*
15. *for each $w \in \hat{W}$ do*
16. *save Z_1 and Z_2 for all its paths in C_w*
17. *sort and filter C_w to $\widehat{C_w}$*
18. *add $\widehat{C_w}$ to $\hat{C}$*
19. *end for*
20. *return $\hat{C}$*

The procedure begins with creating a vast number of paths in the set (H_w). Then, synthetic stochastic demand data is generated through all the time horizon slots and the iterations of the algorithm. While successive DUE assignments are performed to assign each demand data to the network. Travel times for both links and paths are continuously estimated and recorded. The performance of each path considering Z_1 & Z_2 is calculated numerically as in steps (13, 14). The expected travel time of a path is calculating by averaging all times resulted through different iterations, whereas the cumulative distribution is drawn to predict the time that would not be delayed over with a risk equals to the *r.p*.

At last, the generated paths need to be filtered (ranked) to produce the Pareto-set. It may require that each path is compared with every other solution in the set (H_w) for all the O/D pairs to find out if it is dominated or not. The complexity of the sorting step is expected to be large as a general methodology is being targeted with validation for real

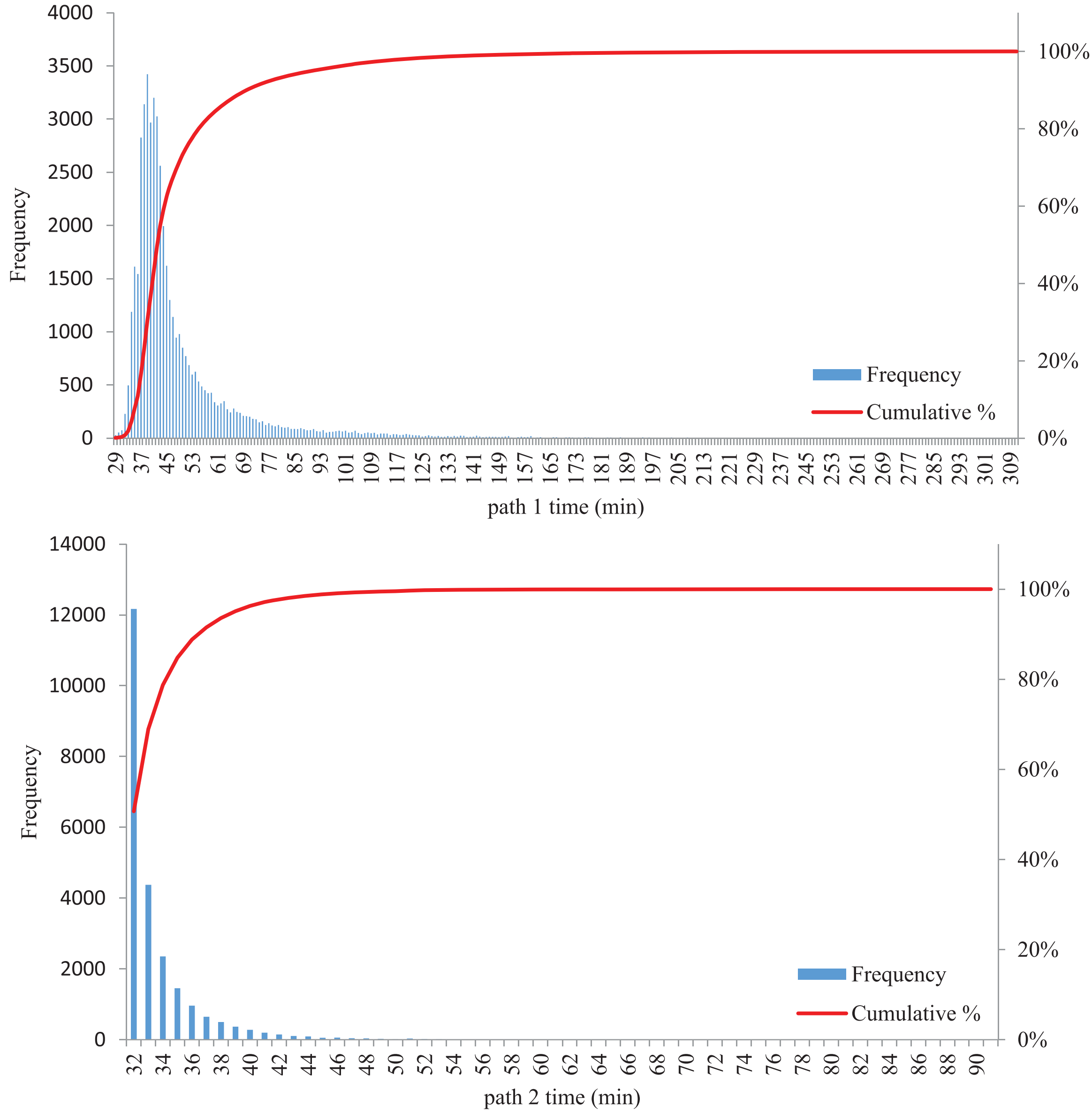


FIGURE 7. Path 1 & path 2 time frequency distribution.

size networks [2]. Therefore, we recommend NSGA-II as a fast sorting algorithm for that purpose. It is an evolutionary algorithm based on Genetic Algorithm (GA) operators, which are developed in [48]. It expedites the process of sorting and filtering when the typical sorting technique takes a long time. The algorithm ends with a final set of paths stored in $\hat{C}$.

V. NUMERICAL STUDY

The proposed methodology was applied to a small network first then to medium-large one. The small network of Nguyen–Dupuis is used to clearly demonstrate the performance of the solution method. Then, the real size network of Austin is used to validate the generality of it. The algorithms were run on PC with Intel Core I7, 2.8 GHz processor and 16 GB of RAM.

A. NGUYEN–DUPUIS NETWORK

It is a fully reported small network in the literature that consists of 13 nodes and 19 links, as shown in Fig. 4. The physical characteristics of the network's links are given in Table 1. The time horizon Θ is selected to be 2 hrs with $\delta = 5$ min (i.e., 24-time steps). The prior demand information is presented in Table 2 whereas total demand (U_τ) time-dependent variation with respect to its mean and standard deviation are presented in Fig. 5. Note that we considered $\gamma_w = 0.2d_0^w$ for all demand pairs and $\kappa = 0.01$ for the assignment convergence. The *r.p.* is taken 5% for the whole analysis.

The path generation algorithm in steps *a* to *c* is conducted to produce the prior path. Then, the final algorithm is performed with 1000 generations for each time slot. The results are reported in Table 3. There is no restriction on ρ since

TABLE 2. Prior demand information of the Nguyen–Dupuis network.

$\hat{W}$	O/D	K	D_0
1	1/2	0.2	20
2	1/3	0.4	40
3	4/2	0.3	30
4	4/3	0.1	10

TABLE 3. Nguyen–Dupuis network paths structure.

$\hat{W}$ (o/d)	Path no. q	Path no. Λ	T_0 min	ρ	Node sequence	Z_1 min[4]	Z_2 min[4]	STD[3]
1/2	N[1]	1	29	1.00	1 5 6 7 8 2	48	88	21.46
	N	2	32	1.10	1 12 8 2	34	39	3.03
	O[2]	3	33	1.14	1 5 6 7 11 2	52	96	22.03
	O	4	30	1.03	1 5 6 10 11 2	51	91	22.84
	O	5	41	1.41	1 5 9 10 11 2	53	90	20.28
	O	6	35	1.21	1 12 6 7 8 2	48	76	23.81
	O	7	39	1.34	1 12 6 7 11 2	52	85	24.33
	O	8	36	1.24	1 12 6 10 11 2	51	79	25.15
1/3	N	1	29	1.00	1 5 6 10 11 3	50	81	23.15
	O	2	35	1.21	1 12 6 10 11 3	50	69	25.40
	O	3	32	1.10	1 5 6 7 11 3	51	86	22.33
	O	4	40	1.38	1 5 9 10 11 3	52	80	20.54
	O	5	36	1.24	1 5 9 13 3	48	63	19.65
	O	6	38	1.31	1 12 6 7 11 3	51	74	24.72
4/2	N	1	31	1.00	4 5 6 7 8 2	42	80	8.81
	N	2	37	1.19	4 9 10 11 2	40	71	5.94
	O	3	35	1.13	4 5 6 7 11 2	46	59	9.52
	O	4	32	1.03	4 5 6 10 11 2	45	83	10.21
	O	5	43	1.39	4 5 9 10 11 2	48	83	7.97
4/3	N	1	31	1.00	4 5 6 10 11 3	44	73	10.88
	N	2	32	1.03	4 9 13 3	35	44	4.29
	O	3	34	1.10	4 5 6 7 11 3	45	79	9.42
	O	4	42	1.35	4 5 9 10 11 3	47	73	7.69
	O	5	38	1.23	4 5 9 13 3	42	55	6.63
	O	6	36	1.16	4 9 10 11 3	39	61	5.42

[1] None overlapped route [2] Overlapped route [3] Standard deviation [4] Minutes

the network is small. For analysis presentation, let us select the 1/2 node pair with the largest number of generated paths. The Box Plot of path 1 variation through the 24-time slots is drawn in Fig. 6. This variation corresponds with the demand variation in Fig.5. Paths 1 and 2, as the non-overlapped, time-frequency distribution is depicted in Fig. 7. Path 1 distribution resembles log –norm distribution whereas path 2 approaches the exponential distribution. Path 2 dominates path 1 and other paths in 1/2 (o/d) set due to its structure which obviates the demand from other paths. The segments 12-8-2 carry only 1/2 demand which makes the path to achieve lower mean (Z_1) with lesser risk time (Z_2). The path case corresponds to ring routes that deliver more reliability in a congested network. Whereas the 1/3 demand node pair has one non-overlapped

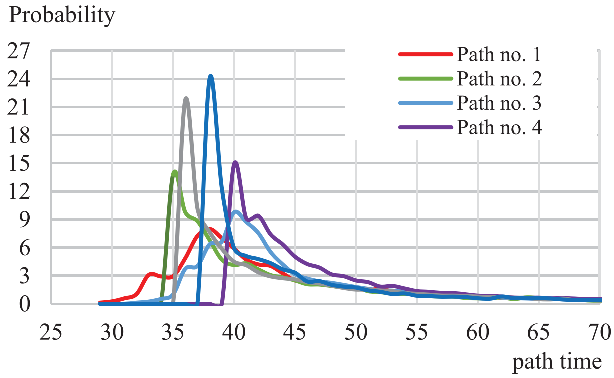


FIGURE 8. Paths time probability distribution for 1/3 node pair.

route and five other paths overlapped with it. In Fig. 8, the probability distribution of these six paths are drawn in which all paths distributions overlapped with large portions. There is no salient dominance among the paths except path 5, which is slightly better.

B. SIOUX FALLS NETWORK

It is a real network that consists of 24 vertices, 76 links, see Fig. 9. All vertices are serving as o/d node pairs with total trips equal to 360600 during the peak hour. The network details and corresponding data are found in a repository called

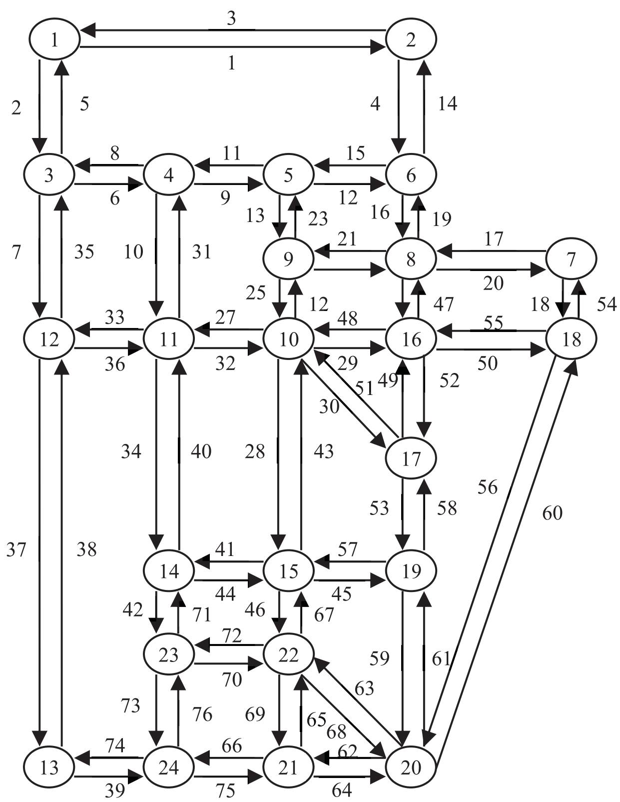


FIGURE 9. Sioux Falls Network structure.

Transportation Networks for Research Core Team (https://github.-com/bstabler/TransportationNeworks).

To confine the analysis, we picked the farthest node pair of 1/20 with $T_0 = 23$ min. The path generation algorithm is run premlinary with $\rho = \infty$ resulting in 3165 paths which are not practical number to consider for routing. Then, it is run with different ρ values ranges between 1 to 2. Fig. 10 depicts the number of generated paths corresponding to each ρ value. At $\rho = 1.5$, a number of 39 paths are produced for the analysis. The Pareto optimal solutions are presented in Fig. 11, where 5 solutions are found to be non-dominated.

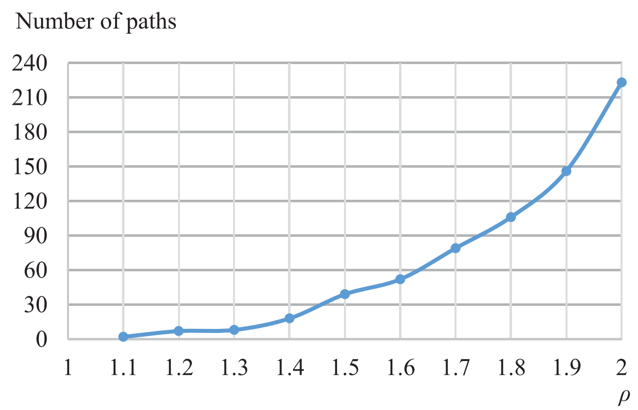


FIGURE 10. Number of paths versus circuitous factor ρ.

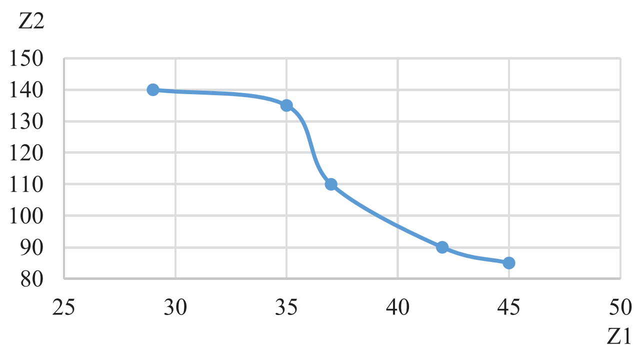


FIGURE 11. Z1 versus Z2 in minutes for the Sioux Falls network.

VI. CONCLUSION

This study proposes a new pre-routing technique for transportation networks that are stochastic in nature. It depends on simulating the network demand variation over the analysis time horizon to examine a set of pre-generated paths. The time horizon is divided into time slots and, at each time slot, there would be stochastic (random) values for the node pairs' demand by which the network is assumed to experience a temporary equilibrium. All trips start and end before the time horizon ends, while the proposed routing decisions do not affect the existing network equilibrium. The methodology managed to produce the expected path times and the reliability of each path for the tested networks. A probability distribution is also drawn from 24000 generations that simulate the different network traffic loading conditions. The algorithm is suitable for any network size since it depends on off-line learning. It managed to reduce the selection from 3165 elementary paths of a node pair in the Sioux Falls Network to just 5 non-dominated paths. This work opens the gate for further investigation using the proposed framework. Various routing algorithms besides different assignment techniques could be examined. Also, the demand assumption could be extended to incorporate a variety of probability distribution functions. Even the demand variation through the time horizon could be replaced with the real recorded historical distribution. Also, further research may focus on enhancing that prior probability distribution by incorporating one of machine learning tools and online information like GPS tracing and traffic sensors counting to produce more reliable posterior probability.

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MAHMOUD OWAIS received the M.S. degree in transportation planning and the Ph.D. degree in transit planning from the University of Assiut, in 2011 and 2014, respectively.

He is currently an Associate Professor of transportation planning and traffic engineering with the Civil Engineering Department, Faculty of Engineering, Assiut University. He published some studies in well-reputed journals regarding transit planning and traffic sensor problems. He was nominated as a Reviewer by the IEEE Transactions on Intelligent Transportation Systems, IEEE Access, and the *European Journal of Operational Research*.

ABDULLAH ALSHEHRI received the Ph.D. degree in management of projects from The University of Manchester, U.K., in 2013. He is currently the Head of the Civil and Environmental Engineering Department, Majmaah University, Majmaah, Saudi Arabia. His research interest is in project management topics, such as scheduling, control, networks, and transportation.